Application of Compressed Sensing to DRM Channel Estimation

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***Abstract*—In order to reduce the pilot number and improve the spectral efficiency, recently emerged compressed sensing (CS) technique is applied for digital broadcast channel estimation. According to the six channel profiles of the ETSI digital radio mondiale (DRM) standard, the subspace pursuit (SP) algorithm is employed for the delay spread and attenuation estimation of each path in the case where the channel profile is identified and the multipath number is known beforehand. The stop condition for SP is that the estimated sparsity equals the multipath number. For the case where the multipath number is unknown, the orthogonal matching pursuit (OMP) algorithm is employed for channel estimation, while the stop condition is that the estimation satisfies the level of the noise variance. Simulation results show that with the same number of pilots, CS algorithms with randomly placed pilots outperform traditional cubic-spline-interpolation-based least square (LS) channel estimation. SP is also demonstrated to be better than OMP when the multipath number is known as *a priori*.**

I. Introduction

Compressed sensing (CS) has recently emerged as a collection of principles and methodologies which enables efficient reconstruction of sparse signals from relatively few linear measurements [1], [2]. We roughly divide CS algorithms into two classes, including greedy algorithms and convex optimization algorithms. Greedy algorithms make a sequential locally optimal choice in an effort to determine a globally optimal solution. They are matching pursuit (MP) [3], orthogonal matching pursuit (OMP) [4], subspace pursuit (SP) [5] and many other variants [6], [7], [8]. Compared to convex optimization algorithms, their computational complexity is much lower, which indicates they are more appropriate for practical applications.

Recently, MP has been applied for pilot assisted channel estimation in orthogonal frequency division multiplex (OFDM) systems [9], [10]. It is demonstrated that the pilot number can be substantially reduced compared to the traditional interpolation based least square (LS) method while the performance of channel estimation is similar. Especially for the time-varying channel where channel estimation should be frequently carried out, CS algorithms are capable of saving a large number of pilots and improving the spectral efficiency. It is also beneficial for multi-input multi-output (MIMO) systems to employ CS algorithms since the pilots increase linearly with the number of transmit antennas. Although MP can find an approximated solution from the overcomplete dictionary with asymptotic convergence, the shortcoming lies in the fact that it may select the same column several times and lower the efficiency. It is necessary to adapt more powerful CS algorithms for relative applications.

In this paper, we apply OMP and SP for the digital broadcast channel estimation. According to the ETSI digital radio mondiale (DRM) standard, six channel profiles are classified. For the case where the multipath number is unknown, we apply OMP for channel estimation. For the case where channel profile is identified and the multipath number is known beforehand, we apply SP for the delay spread and attenuation estimation of each path. In both cases, we formulate the pilot assisted OFDM frequency domain channel estimation as a sparse recovery problem. We also compare their performance with traditional interpolation based LS method with differently placed pilots.

The remainder of the paper is organized as follows. Section II formulates the DRM channel estimation. Section III and Section IV introduce the OMP and the SP algorithm. In Section V, the pilot placement and pilot number are discussed. Simulation results are presented in Section VI, and finally section VII concludes the paper.

The notation used in this paper is according to the convention. Symbols for matrices (upper case) and vectors (lower case) are in boldface. $(\cdot)^H$, $|\cdot|$, $\|\cdot\|_1$ $\|\cdot\|_2$, $\mathbb{C}$, $\mathbb{R}$, $diag\{\cdot\}$, $\boldsymbol{I}_L$ and $\mathcal{CN}$ denote conjugate transpose (Hermitian), absolute value, ℓ_1-norm, ℓ_2-norm, the set of complex number, the set of real number, diagonal matrix, identity matrix with the dimension L and complex Gaussian distribution, respectively. $\hat{\phi}$ denotes the estimate of the parameter of interest ϕ.

II. Problem Formulation

We model the channel impulse response (CIR) of multipath propagation as

$$h(\tau, t) = \sum_{p=1}^{S} \alpha_p(t)\delta(\tau - \tau_p(t)) \tag{1}$$

where S, α_p and τ_p are the multipath number, the channel coefficient and the delay spread for the p-th propagation path. According to the ETSI DRM standard [11], we have six channel profiles as listed in Tab.I. Channel No.1 is the pure AWGN channel, which is not common in practice. Channels No.2, No.4 and No.5 are two-path channels with increasing delay spread. Both channel No.3 and No.6 are four-path channels where eight parameters should be estimated.

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TABLE I
SIX CHANNEL PROFILES IN THE ETSI DRM STANDARD

Channel	Path 1		Path 2		Path 3		Path 4	
No.	α_1	τ_1 / ms	α_2	τ_2 / ms	α_3	τ_3 / ms	α_4	τ_4 / ms
1	1	0	0	0	0	0	0	0
2	1	0	0.5	1	0	0	0	0
3	1	0	0.7	0.7	0.5	1.5	0.25	2.2
4	1	0	1	2	0	0	0	0
5	1	0	1	4	0	0	0	0
6	0.5	0	1	2	0.25	4	0.0625	6

With block fading channel assumption where the channel parameters are constant over each block and supposing perfect symbol synchronization, we model the equivalent discrete CIR as

$$h(m) = \sum_{p=1}^{S} \alpha_p \delta((m - \tau_p)T_s) \tag{2}$$

where T_s is the sampling interval of the receiver. We notice that the elementary time period T of each OFDM symbol in the DRM standard is no more than 0.1 ms, so T_s should be less than 0.05 ms, which is very small compared to the maximum delay spread and results in a channel with relatively few nonzero taps. Supposing the number of channel taps to be L and S of them nonzero, we define it as an S-sparse channel. Considering an OFDM system, there are totally N subcarriers, among which M subcarriers are selected as pilots, with the positions represented by $Q_1, Q_2, \ldots, Q_M$ $(1 \leq Q_1 < Q_2 < \cdots < Q_M \leq N)$. We denote the transmitted pilots and the received pilots as $X(Q_1), X(Q_2), \ldots, X(Q_M)$ and $Y(Q_1), Y(Q_2), \ldots, Y(Q_M)$, respectively. We have

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{h} + \boldsymbol{\eta} \tag{3}$$

where $\boldsymbol{y} = [Y(Q_1), Y(Q_2), \cdots, Y(Q_M)]^{\mathrm{T}}$, $\boldsymbol{h} = [h(1), h(2), \cdots, h(L)]^{\mathrm{T}}$ and $\boldsymbol{\eta} = [\eta(1), \eta(2), \cdots, \eta(M)]^{\mathrm{T}}$. $\boldsymbol{\eta} \sim \mathcal{CN}(0, \sigma^2 \boldsymbol{I}_M)$. And

$$\boldsymbol{A} = \boldsymbol{Z}\boldsymbol{F} \tag{4}$$

where

$$\boldsymbol{F}_{N_p \times L} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \omega^{Q_1} & \cdots & \omega^{Q_1 \cdot (L-1)} \\ 1 & \omega^{Q_2} & \cdots & \omega^{Q_2 \cdot (L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{Q_M} & \cdots & \omega^{Q_M \cdot (L-1)} \end{bmatrix}$$

is a submatrix selected by the row index $[Q_1, Q_2, \ldots, Q_M]$ and the column index $[0, 1, \ldots, L-1]$ from a standard $N \times N$ Fourier matrix. $\omega = e^{-j2\pi/N}$. $\boldsymbol{Z} = diag\{X(Q_1), X(Q_2), \ldots, X(Q_M)\}$.

If the rows of $\boldsymbol{A}$ are more than its columns, (3) can be solved by the traditional LS method. However, we are more interested in the under-determined case where the rows of $\boldsymbol{A}$ is less than its columns, which means the pilots are less than the unknown channel coefficients. Still we can use

$$H(i) = \frac{Y(i)}{X(i)}, \ i = Q_1, \ Q_2, \ \ldots, \ Q_M \tag{5}$$

first to figure out the channel transfer function (CTF) at the pilot subcarriers and then make linear or cubic spline interpolations for the data subcarriers. Obviously this method will result in large deviations because it doesn't use the sparse condition as *a priori*.

Here we focus on low-complexity greedy CS algorithms and divide the sparse channel estimation problem into two cases. In the first case, the multipath number is unknown, and we apply OMP for channel estimation. For the case where the channel profile is identified and the multipath number is known beforehand, we apply SP for the delay spread and attenuation estimation of each path.

III. MP AND OMP

MP is a sort of algorithm that constructs a sparse solution by iteratively selecting dictionary elements best correlated with the residual part of the signal [3]. If the dictionary is a matrix, the objective of the construction is to find a linear combination of matrix columns closest to the signal. At each step, one column that best correlates with the current residue is added to current selection. Then it updates the residue by projecting it onto the new selection. Here we briefly describe MP and OMP algorithm based on (3).

First we generate a dictionary matrix $\boldsymbol{D}$ from $\boldsymbol{A}$,

$$\boldsymbol{A} = \boldsymbol{D}\boldsymbol{C} \tag{6}$$

where $\boldsymbol{D}$ has the same dimension as $\boldsymbol{A}$ ($\boldsymbol{D} \in \mathbb{R}^{N_p \times L}$) and each column of $\boldsymbol{D}$ is a unit vector (ℓ_2-norm equals one). $\boldsymbol{C}$ is a diagonal matrix with each diagonal element corresponding to the normized coefficient for each column of $\boldsymbol{A}$. Let $\boldsymbol{d}_i$ denote the i-th column of $\boldsymbol{D}$ and $\boldsymbol{R}_k$ denote the residue at the k-th step. So the selected column index at k-th step is

$$l_k = \arg \max_{i \in \{1, \ldots, L\}} | < \boldsymbol{d}_i, \boldsymbol{R}_k > | \tag{7}$$

where $| < \boldsymbol{d}_i, \boldsymbol{R}_k > |$ represents the absolute value of inner product between $\boldsymbol{d}_i$ and $\boldsymbol{R}_k$.

Now starting MP with an initial residue $\boldsymbol{R}_1 = \boldsymbol{y}$, the algorithm evolves by

$$\boldsymbol{R}_k = < \boldsymbol{d}_{l_k}, \boldsymbol{R}_k > \boldsymbol{d}_{l_k} + \boldsymbol{R}_{k+1} \tag{8}$$

and replaces the residue $\boldsymbol{R}_k$ with $\boldsymbol{R}_{k+1}$. Since $\boldsymbol{R}_{k+1}$ is orthogonal to $\boldsymbol{d}_{l_k}$, we have

$$||\boldsymbol{R}_k||_2^2 = |<\boldsymbol{d}_{l_k}, \boldsymbol{R}_k>|^2 + ||\boldsymbol{R}_{k+1}||_2^2 \tag{9}$$

With increasing k ($k = 1, 2, \ldots$), we minimize $||\boldsymbol{R}_{k+1}||_2$ till it satisfies the termination condition

$$||\boldsymbol{R}_{k+1}||_2 \leq \sigma \tag{10}$$

Supposing the iteration ends in K steps, the solution is

$$\boldsymbol{d} = \sum_{i=1}^{K} <\boldsymbol{d}_{l_i}, \boldsymbol{R}_i> \boldsymbol{d}_{l_i} \tag{11}$$

which is a linear combination of previous selected columns. Finally, the estimation of $\boldsymbol{h}$ is

$$\hat{\boldsymbol{h}}_{MP} = \boldsymbol{C}^{-1}\boldsymbol{d} = \boldsymbol{C}^{-1} \cdot \sum_{i=1}^{K} <\boldsymbol{d}_{l_i}, \boldsymbol{R}_i> \boldsymbol{d}_{l_i} \tag{12}$$

Although MP can rapidly find an approximation from overcomplete dictionary with asymptotic convergence, the shortcoming lies in the fact that it may select the same columns several times and thus brings down the efficiency. Hence, OMP [12] is proposed with revision by using residue's orthogonal component for the next iteration. Only the component that is orthogonal with the space spanned by the previous selected columns is preserved. In most literatures, the set containing previous selected columns is called the *active set*. Here we denote the *active set* and its complementary set as I and I^c, respectively. $I \cup I^c = \{1, \ldots, L\}$. Unlike MP always selecting candidate column from $\{1, \ldots, L\}$ as in (7), OMP selects it only from I^c, where the selected column index is

$$l_k = \arg\max_{i \in I^c} |<\boldsymbol{d}_i, \boldsymbol{R}_k>| \tag{13}$$

Then Gram-Schmidt orthogonalization is implemented to remove the component within the space spanned by I.

$$\boldsymbol{u}_k = \boldsymbol{d}_{l_k} - \sum_{i \in I} \frac{<\boldsymbol{d}_{l_k}, \boldsymbol{u}_i>}{||\boldsymbol{u}_i||_2^2} \boldsymbol{u}_i \tag{14}$$

where $\{\boldsymbol{u}_k\}$ is an iteratively generated set that can be thought as a base vector set of the space spanned by I. We initialize $\boldsymbol{u}_1$ to be $\boldsymbol{d}_{l_1}$ and iteratively update the residue by

$$\boldsymbol{R}_k = \frac{<\boldsymbol{R}_k, \boldsymbol{u}_k>}{||\boldsymbol{u}_k||_2^2} \boldsymbol{u}_k + \boldsymbol{R}_{k+1} \tag{15}$$

In this way, OMP adds different columns into the *active set* until the stoping condition (10) is satisfied. Supposing it ends in K steps (usually $K \ll L$), the solution is

$$\boldsymbol{d} = \sum_{i=1}^{K} \frac{<\boldsymbol{R}_i, \boldsymbol{u}_i>}{||\boldsymbol{u}_i||_2^2} \boldsymbol{u}_i \tag{16}$$

And eventually the estimation of $\boldsymbol{h}$ is

$$\hat{\boldsymbol{h}}_{OMP} = \boldsymbol{C}^{-1}\boldsymbol{d} = \boldsymbol{C}^{-1} \cdot \sum_{i=1}^{K} \frac{<\boldsymbol{R}_i, \boldsymbol{u}_i>}{||\boldsymbol{u}_i||_2^2} \boldsymbol{u}_i \tag{17}$$

Compared with MP, OMP is much faster convergent. Even for the large dictionary matrix, the step is countable. It has already been demonstrated by [13], [4], [14] that in some circumstances OMP does succeed at finding the sparsest solution. But the discussion is going on. OMP works in greedy way, which determines the final solution is essentially suboptimal, not global optimal. Besides, OMP always adds a new column to the *active set*, but never removes out-dated columns from the *active set*. When a selection error occurs, the iteration will continue till reaching the end without correcting them adaptively. Although several revised versions of OMP have been proposed against these disadvantages [6], [7], [8], OMP is still one of the best candidates for practical applications because of its reasonable tradeoff between the performance and complexity.

IV. SP

In the SP algorithm, the S columns selection is iteratively refined from $\boldsymbol{A}$ until the stop condition is satisfied [5]. Although it is still developed based on the greedy rule, SP allows new columns to enter into as well as to leave the *active set*. S columns are simultaneously selected rather than only one column as in MP and OMP. In this way, the subspace spanned by S columns is tracked down.

Algorithm 1 Subspace Pursuit Algorithm

Input: $\boldsymbol{A}$, $\boldsymbol{y}$, S

Initialization:

Normalize each column of matrix $\boldsymbol{A}$ with a diagonal coefficient matrix $\boldsymbol{C}$ so that $A = \boldsymbol{D} \cdot \boldsymbol{C}$

$\hat{I} = \{$ S indices corresponding to S columns of $\boldsymbol{D}$ on which $\boldsymbol{y}$ has the largest projections $\}$

$\boldsymbol{R}_1 = \text{resid}(\boldsymbol{y}, \boldsymbol{D}_{\hat{I}})$

Iteration: $k = 1, 2, \ldots$

If $\boldsymbol{R}_k = 0$, quit the iteration; otherwise continue

$I' = \hat{I} \bigcup$ $\{$ S indices corresponding to S columns of $\boldsymbol{D}$ on $\boldsymbol{R}_k$ has the largest projections $\}$

Get $\boldsymbol{D}_{I'}^{\dagger}$ and figure out $\boldsymbol{x} = \boldsymbol{D}_{I'}^{\dagger}\boldsymbol{y}$

$I_k = \{$ S indices corresponding to S elements with the largest absolute value in $\boldsymbol{x}$ $\}$

$\boldsymbol{R}_{k+1} = \text{resid}(\boldsymbol{y}, \boldsymbol{D}_{I_k})$

If $||\boldsymbol{R}_{k+1}||_2 > ||\boldsymbol{R}_k||_2$, quit the iteration

otherwise, let $\hat{I} = I_k$

increase k by one and continue the iteration

Output:

One estimate vector $\hat{\boldsymbol{x}}$ is yielded with nonzero element indexed by $\hat{I}$ satisfying $\hat{\boldsymbol{x}}_{\hat{I}} = \boldsymbol{D}_{\hat{I}}^{\dagger}\boldsymbol{y}$

The final result is $\hat{\boldsymbol{h}}_{sp} = \boldsymbol{C}^{-1}\hat{\boldsymbol{x}}$

The SP algorithm solves the problem of (3) and works as follows. First, it begins with the same step as (6) to normalize each column of $\boldsymbol{A}$. Then, it finds S columns from $\boldsymbol{D}$ on which $\boldsymbol{y}$ has the S largest projections, and stores their indices into a set $\hat{I}$. Meanwhile the residue R_1 is also obtained. The projection of $\boldsymbol{y}$ onto each column vector is defined as the absolute value of their inner product which is similar to (7). Here we introduce the definition of the residue since it is different from MP and OMP.

Definition 1: If $\boldsymbol{D}^H\boldsymbol{D}$ is invertible, the residue of $\boldsymbol{y}$ on the matrix $\boldsymbol{D}$ is defined as

$$\boldsymbol{R} = \boldsymbol{y} - \boldsymbol{D}\boldsymbol{D}^{\dagger}\boldsymbol{y} \tag{18}$$

where

$$\boldsymbol{D}^{\dagger} = (\boldsymbol{D}^H\boldsymbol{D})^{-1}\boldsymbol{D}^H \tag{19}$$

denotes the pseudo inverse of $\boldsymbol{D}$. We simply write it as $\boldsymbol{R} = \mathrm{resid}(\boldsymbol{y}, \boldsymbol{D})$.

In the last step of initialization, we get an submatrix $\boldsymbol{D}_{\hat{I}}$ from $\boldsymbol{D}$ by index set $\hat{I}$ and then figure out the residue $\boldsymbol{R}_1 = \mathrm{resid}(\boldsymbol{y}, \boldsymbol{D}_{\hat{I}})$. After that we start the iteration. Another S indices corresponding to S columns of $\boldsymbol{D}$ on which $\boldsymbol{R}_k$ has the largest projections are selected and added to the previous S columns set, forming $\boldsymbol{I}'$. The number of columns in $\boldsymbol{I}'$ may be less than $2S$ since it's possible to have overlap between two selections. Then we figure out $\boldsymbol{D}_{I'}^{\dagger}$ and a vector $\boldsymbol{x} = \boldsymbol{D}_{I'}^{\dagger}\boldsymbol{y}$, from which we choose S indices corresponding to S elements with the largest absolute value. The *active set* I_k at step k is obtained and a new residue $\boldsymbol{R}_{k+1} = \mathrm{resid}(\boldsymbol{y}, \boldsymbol{D}_{I_k})$ is also yielded. If

$$||\boldsymbol{R}_{k+1}||_2 > ||\boldsymbol{R}_k||_2 \tag{20}$$

the iteration is terminated because it indicates that the residue can't be smaller. Otherwise we replace $\hat{I}$ with I_k, increase k by one and continue the iteration. It's noticed that at the start point of every iteration, current residue $\boldsymbol{R}_k$ is on focuss. If $\boldsymbol{R}_k$ is zero, we will also terminate the iteration.

Finally one estimated vector $\hat{\boldsymbol{x}}$ is yielded. The nonzero locations of $\hat{\boldsymbol{x}}$ are indexed by $\hat{I}$ and satisfy $\hat{\boldsymbol{x}}_{\hat{I}} = \boldsymbol{D}_{\hat{I}}^{\dagger}\boldsymbol{y}$. The output is

$$\hat{\boldsymbol{h}}_{SP} = \boldsymbol{C}^{-1}\hat{\boldsymbol{x}} \tag{21}$$

which is the estimated CIR for (3).

V. Pilot Placement

Recent advances in CS show that under noiseless condition as $\boldsymbol{\eta} = 0$ in (3), $\boldsymbol{h}$ can be recovered from $\boldsymbol{y}$ with high accuracy when $\boldsymbol{A}$ satisfies the restricted isometry property (RIP) [15].

Definition 2: $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ satisfies RIP if

$$(1-\delta)||\boldsymbol{h}||_2^2 \le ||\boldsymbol{A}\boldsymbol{h}||_2^2 \le (1+\delta)||\boldsymbol{h}||_2^2 \tag{22}$$

holds for all S-sparse vectors $\boldsymbol{h} \in \mathbb{R}^n$ ($||\boldsymbol{h}||_0 \le S$). Defining δ_s as the infimum of all possible δ that satisfies (22), we say $\boldsymbol{A}$ satisfies RIP(S, δ_s).

It can be easily obtained from (22) that

$$1-\delta \le \lambda_{\min} \le \lambda_{\max} \le 1+\delta \tag{23}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues of $\boldsymbol{A}^H\boldsymbol{A}$, respectively. Since (22) requires all S-sparse vectors to be satisfied, it's difficult to decide δ_s and thus difficult to verify (23). So far there's no method to test in polynomial time whether a given matrix satisfies RIP. Nevertheless, recent works by some mathematicians [15], [16] have shown that certain matrices satisfy RIP with high probability, i. e. random matrix from Fourier ensemble, Gaussian ensemble and binary ensemble.

Concerning our channel estimation problem (3), we have to decide both the number and positions of pilots. For simplicity, we suppose all pilots have the same magnitude and phase, which means $\boldsymbol{Z}$ in (4) is an identity matrix and then $\boldsymbol{A}$ becomes a Fourier submatrix. According to the RIP, we can randomly place the pilot symbols, which in essence is the random-row selection from standard Fourier matrix. On the other hand, the number of pilots has to be at least twice of the sparsity S so as to exactly reconstruct $\boldsymbol{h}$. And it's further demonstrated by [17] that if the pilot number is

$$N_p = O\Big((CtS\log L)\log(CtS\log L)\log^2 S\Big) \tag{24}$$

where C and t ($t > 1$) are two parameters, the matrix $\boldsymbol{A}$ will satisfy RIP with probability at least $1-5e^{-Ct}$. Therefore, in our application we will synchronously generate pseudo-random pilot placement in the same way for the transmitter and the receiver. Although it's well known that the best pilot placement for mobile OFDM systems is equally spaced [18] and it is shown in [19] that the optimal pilot sequences are equipowered, equispaced and phase shift orthogonal using LS channel estimation, we here employ CS algorithms instead of the LS estimator. According to the theoretical conclusion of RIP, we will use the random pilot placement.

VI. Simulation results

In our simulations, we set OFDM parameters according to the robustness mode B in the DRM standard as shown in Tab.II. The channel profile is set to be channel No.3 as described in Tab.I. We define the mean square error (MSE) as

$$\mathrm{MSE}\{\hat{\boldsymbol{h}}\} = \frac{||\hat{\boldsymbol{h}} - \boldsymbol{h}||_2^2}{||\boldsymbol{h}||_2^2} \tag{25}$$

where $\hat{\boldsymbol{h}}$ is the estimate of $\boldsymbol{h}$.

TABLE II
System parameters

FFT length	$N_{FFT} = 256$
Used subcarriers	$N = 206$
Guard interval	$N_g = 64$
OFDM symbol period	$T_s = 26.7$ms
OFDM sample period	$T = 83.3$us
Number of pilots	$M = 12$
Number of channel multipaths	$S = 4$
CIR length	$L = 30$
Modulation	QPSK

Firstly, we suppose the multipath number and the channel profile are unknown to the receiver. We compare the MSE for OMP and LS. Equal-spaced pilot placement and random pilot placement are employed for LS and OMP, respectively. In Fig.1, OMP with M = 12 is even better than cubic-spline-interpolation-based LS with M = 69 for SNR> 20dB, while their spectral efficiency is 94% and 66%, respectively. So OMP achieves more accurate channel estimation than LS while the pilot symbols are less used.

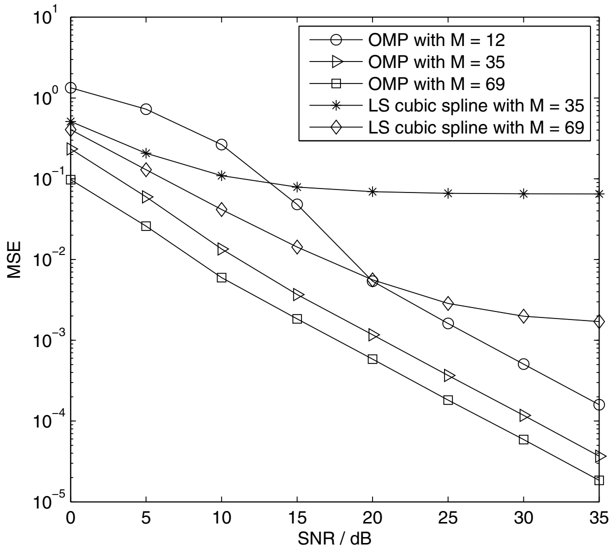

Fig. 1. MSE comparisons for OMP and LS

Then we suppose the multipath number is known as *a priori* to the receiver where we apply OMP and SP for the delay spread and attenuation estimation of each path. When M = 10, SP is clearly superior to OMP. When M = 12, SP is still a little better than OMP. The reason that SP performs better than OMP lies in the nature of each algorithm. At each iterative step, OMP always greedily selects one column vector while SP selects several columns in batch. The possibility to correctly find one column with one selection is much lower than with batch selection. Besides, once OMP selects one column into the *active set*, it never removes it, which means one error selection occurs, it will never be corrected in the later steps.

VII. Conclusion

In this paper we formulate the DRM channel estimation as a sparse recovery problem. It is shown that the CS algorithms are much better than the traditional LS method. If the multipath number is known beforehand, SP outperforms OMP. Future research will concentrate on the development of lower complexity CS algorithms with quantitative comparisons.

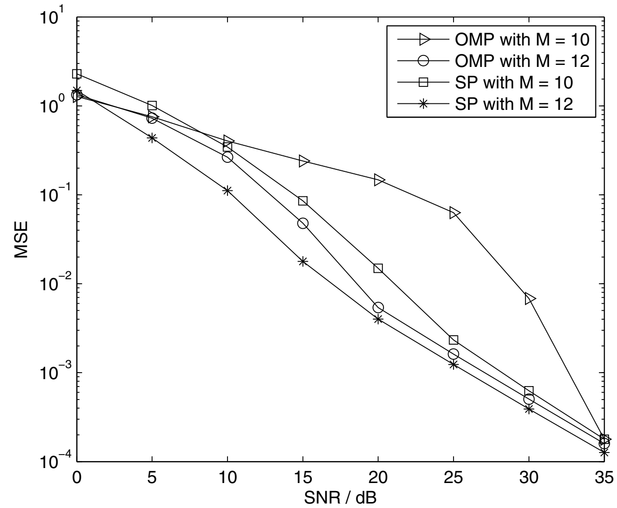

Fig. 2. MSE comparisons for OMP and SP

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