Research Article

Antenna placement optimisation for compressed sensing-based distributed MIMO radar

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Abstract: Since a better localisation performance is achieved by exploiting the spatial diversity in the distributed multiple-input and multiple-output (MIMO) radar systems, the authors consider the problem of further improving this performance by optimising the antenna placement in this study. In the localisation processes, a novel compressed sensing-based method is proposed to exploit the target sparsity, where an over-complete dictionary matrix containing all the possible echo waveforms from the discretised area is first established. In addition, to further improve the localisation performance, a novel iterative method is proposed to minimise the mutual coherence of the dictionary matrix by optimising the antenna placement of both transmitters and receivers. Moreover, the theoretical distribution of the mutual coherence is derived, and an asymptotic performance with increasing the number of antennas is also provided. Simulation results demonstrate the localisation performance improvement for multiple targets by optimising the antenna placement in the distributed MIMO radar.

1 Introduction

In multiple-input and multiple-output (MIMO) radar, a better performance of target detection and localisation can be achieved by the antenna diversity and the independently transmitted waveforms [1–3]. Based on the distance between antennas, the MIMO radar can be categorised into colocated and distributed MIMO radar:

- i. In the colocated MIMO radar, the antennas are close to each other, and the waveform diversity is exploited to improve the target detection and estimation performance [4, 5].
- ii. In the distributed MIMO radar, the antennas are widely separated, and the spatial diversity of target's radar cross section is exploited to improve the target detection performance [6, 7]; and moreover, an improved performance of target localisation can be achieved by integrating the delays between any pair of transmitters (TXs) and receivers (RXs) [3, 8–11].

Therefore, in this paper, we consider the problem of localising multiple targets by the distributed MIMO radar. According to compressed sensing (CS) theory, a sparse signal can be reconstructed from far fewer measurements than that required in the conventional sampling theory [12, 13]. To exploit the target sparsity in the detection area, a CS-based MIMO radar system has been proposed [14], where the improved localisation performance is achieved with far fewer measurements than that required in the traditional MIMO radar [15].

To improve the target localisation performance, methods have also been proposed to optimise the MIMO radar system. For example, the received waveforms are jointly processed in [16, 17], and the joint estimation of both target position and velocity has been proposed in [18]. In addition, the CS-based MIMO radar has also been optimised to improve the target localisation performance. For example, the measurement matrix [19], the power allocation, and the transmitted waveforms [14] are all optimised to decrease the mutual coherence of the sensing matrix.

In this paper, the localisation for multiple stationary targets is addressed in the distributed MIMO radar system. The detection area is first discretised into grids, and an over-complete dictionary matrix containing the echo waveforms from all grids is built. Then, we formulate it as a sparse reconstruction problem, where the elements in the support set correspond to the target positions. Therefore, by exploiting the target sparsity, the CS-based methods are adopted to reconstruct the sparse vector and localise the targets. In addition, we obtain the theoretical distribution of the mutual coherence with the random placement of antennas, and analyse the asymptotical property with the infinite number of antennas. Since the target localisation performance depends on the placement of TXs and RXs in the distributed MIMO radar [20, 21], the target localisation can also be improved benefiting from the geographical deployment [22, 23]. So we propose an iterative method to optimise the placement of both TXs and RXs, and to further improve the performance of target localisation by minimising the mutual coherence of the dictionary matrix. Moreover, the performance improvement is also measured by the theoretical formulations

In the remainder of this work, the system model of distributed MIMO radar is given in Section 2. The CS model for target localisation is provided in Section 3. Section 4 proposes an iteration method to optimise the antennas placement and minimise the mutual coherence of dictionary matrix. The theoretical distribution of the mutual coherence with the random antennas placement is analysed in Section 5. Section 6 shows the simulation results and Section 7 concludes this paper.

The notations used in this work are defined as follows. Symbols for vectors (lower case) and matrices (upper case) are in bold face. $\mathbb{N}_{+}, (\cdot)^{\mathrm{H}}, (\cdot)^{\mathrm{T}}, \operatorname{diag}\{\cdot, I_{Q}, \mathscr{CN}(0, \mathbf{R}), \|\cdot\|_{0}, \|\cdot\|_{1}, \|\cdot\|_{2}$ and vec $\{\cdot, \cdot\}$ denote the natural number set, the conjugate transpose (Hermitian), the transpose, the diagonal matrix, the $Q \times Q$ identity matrix, the complex Gaussian distribution with zero mean and covariance being \mathbf{R} , the ℓ_{0} norm, the ℓ_{1} norm, the ℓ_{2} norm and the vectorisation of a matrix, respectively.

2 System model

As shown in Fig. 1, the distributed MIMO radar is adopted to localise multiple stationary targets, where the number of TXs, RXs and targets are *M*, *N* and *K*, respectively. All TXs, RXs and targets are widely separated in the area $(0, x_0) \times (0, y_0)$. Orthogonal waveforms are transmitted independently by the TXs, and the waveform of the *m*th TX is denoted as Re $\{s_m(t) e^{j2\pi f_c t}\}$, where $s_m(t)$ $(0 \le t \le T)$ denotes the complex baseband waveform, f_c denotes the



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Fig. 1 Distributed MIMO radar system



Fig. 2 Discretised detection area

carrier frequency, T denotes the pulse duration, and t denotes the continuous time.

The waveform, which is transmitted by the *m*th TX, echoed from the *k*th target and received by the *n*th RX, can be represented as

$$y_{m,n,k}(t) = \operatorname{Re}\left\{\alpha_k s_m(t - \tau_{m,n,k}) e^{j2\pi f_c(t - \tau_{m,n,k})}\right\},$$
(1)

where α_k denotes the scattering coefficient of the *k*th target. Since the placement optimisation for the MIMO radar is mainly considered in this paper, for the simplification, we assume that the scattering coefficients of the different targets are different, but keep the same for different view angles of the same target. This assumption has also been considered in [15, 24–26]. $\tau_{m,n,k}$ denotes the delay of the *k*th target, and can be expressed as

$$\tau_{m,n,k} = \frac{1}{c} \Big(\| \boldsymbol{p}_{T,m} - \boldsymbol{t}_k \|_2 + \| \boldsymbol{p}_{R,n} - \boldsymbol{t}_k \|_2 \Big),$$
(2)

where $p_{T,m} \in \mathbb{R}^2$, $p_{R,n} \in \mathbb{R}^2$ and $t_k \in \mathbb{R}^2$ denote, respectively, the positions of the *m*th TX, the *n*th RX and the *k*th target, and *c* denotes the speed of electromagnetic wave. Therefore, collecting the echo waveforms from all targets and TXs, the received signal of the *n*th RX can be written as

$$y_n(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} y_{m,n,k}(t).$$
(3)

At each RX, there are *M* matched filters for all the orthogonally transmitted waveforms, where the matched filter for the *m*th transmitted waveform can be expressed as $h_m(t) = s_m^{\rm H}(T-t)$. Then, after down converting and passing through all matched filters, the output of the *m*th matched filter at the *n*th RX can be written as

$$y'_{n,m}(t) = \sum_{k=0}^{K-1} \alpha_k e^{-j2\pi f_c \tau_{m,n,k}} \int s_m(t' - \tau_{m,n,k}) h_m(t-t') dt'.$$
 (4)

For the *n*th RX, by assuming that the *k*th target is on the grid, we sample the integration waveform at $t = T + \tau_{m,n,k}$, where $\tau_{m,n,k}$ denotes the delay of the *k*th target. The sampled signal of the *m*th matched filter is

$$y_{m,n} = \sum_{k=0}^{K-1} a_k e^{-j2\pi f_c \tau_{m,n,k}} = \boldsymbol{\phi}_{m,n}^{\mathrm{T}} \boldsymbol{\alpha}, \qquad (5)$$

where the transmitted power is $\int_0^T s_m^H(t) s_m(t) dt = 1$, the scattering coefficient vector is $\boldsymbol{\alpha} \triangleq [\alpha_0, \alpha_1, \dots, \alpha_{K-1}]^T$, and the phase vector caused by delay is

$$\boldsymbol{\phi}_{m,n} \triangleq \left[e^{-j2\pi f_c \tau_{m,n,0}}, e^{-j2\pi f_c \tau_{m,n,1}}, \dots, e^{-j2\pi f_c \tau_{m,n,(K-1)}} \right]^{\mathrm{T}}.$$
 (6)

Then, collecting sampled signals $y_{m,n}$ from all RXs, a vector can be obtained

$$\mathbf{y} \triangleq \operatorname{vec}\{\mathbf{Y}\} = \mathbf{\Phi}\boldsymbol{\alpha},\tag{7}$$

where the *m*th row and *n*th column of *Y* is

$$y_{m,n}$$
 and $\Phi \triangleq \begin{vmatrix} \phi_{0,0}^{T} \\ \vdots \\ \phi_{(M-1),0}^{T} \\ \vdots \\ \phi_{(M-1),(N-1)}^{T} \end{vmatrix}$

Finally, with the additive white Gaussian noise (AWGN) $n \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{MN})$, where σ_n^2 denotes the noise variance, the received vector can be written as

$$r = \Phi \alpha + n \,. \tag{8}$$

3 Localisations for multiple targets

In this paper, we use the CS-based methods to localise multiple targets. First, as shown in Fig. 2, we discretise the target area $(0, x_0) \times (0, y_0)$ into $P \times Q$ grids, where $x_0 = P\Delta$, $(P \in \mathbb{N}_+)$, $y_0 = Q\Delta$, $(Q \in \mathbb{N}_+)$, and Δ denotes the range resolution. Then, an over-complete dictionary matrix Ψ can be obtained by collecting the sampled signals from all the grids. The dictionary matrix Ψ is defined as

$$\boldsymbol{\Psi} \triangleq \left[\boldsymbol{\psi}_{0}, \boldsymbol{\psi}_{1}, \dots, \boldsymbol{\psi}_{PQ-1}\right] \in \mathbb{C}^{MN \times PQ},$$
(9)

where $PQ \gg MN$. $\varepsilon_{;a}$ ($0 \le a \le PQ - 1$) denotes the *a*th column of Ψ , and indicates the sampled signals echoed from the *a*th grid point. Therefore, $\varepsilon_{;a}$ can be written as

$$\Psi_{a} = \left[e^{-j2\pi f_{c}\tau_{0,0,a}} \dots e^{-j2\pi f_{c}\tau_{0,N-1,a}} \dots e^{-j2\pi f_{c}\tau_{m,0,a}} \dots e^{-j2\pi f_{c}\tau_{M-1,N-1,a}} \right]^{T} (10)$$

$$\in \mathbb{C}^{MN \times 1},$$

where $\tau_{m,n,a}$ is the delay from the *m*th TX to the *a*th grid point and received by the *n*th RX, and $\tau_{m,n,a}$ can be obtained from (2) by replacing the target position with that of grid point.

The received signal in (8) can be rewritten as

$$\boldsymbol{r} = \boldsymbol{\Psi} \boldsymbol{x} + \boldsymbol{n},\tag{11}$$

IET Radar Sonar Navig., 2017, Vol. 11 Iss. 2, pp. 285-293 © The Institution of Engineering and Technology 2016 where $\mathbf{x} \in \mathbb{C}^{PQ \times 1}$ denotes a sparse vector. The number of non-zero entries in \mathbf{x} equals to the number of targets, the non-zero entries indicate the target scattering coefficients a, and the index of non-zero entries correspond to the targets positions. For example, the scattering coefficient of the *k*th target is a_k and the target position t_k is on the *b*th grid. Then, the *b*th entry $(0 \le b \le PQ - 1)$ of \mathbf{x} , denoted as x_{b} , is a_k , i.e. $x_b = a_k$.

Therefore, the target locations and scattering coefficients can be estimated by reconstructing the sparse vector \mathbf{x} from the received signal \mathbf{r} . Then, the following CS optimisation problem can be adopted:

$$\min_{\mathbf{x}} \| \mathbf{x} \|_{0} \quad \text{s.t.} \| \mathbf{r} - \Psi \mathbf{x} \|_{2} \le \epsilon, \tag{12}$$

where ε is used to control the reconstruct accuracy and is usually set as $\varepsilon = \sigma_n$, and ℓ_0 counts the number of non-zero elements. Then, we can adopt the CS-based methods to reconstruct the sparse vector \mathbf{x} and estimate the target locations and scattering coefficients. The CS algorithms can be categorised into two types including the ℓ_1 optimisation methods and the greedy algorithms [27, 28].

In the ℓ_1 optimisation methods, the ℓ_0 norm problem in (12) is relaxed to a convex problem by the ℓ_1 norm

$$\min_{\mathbf{x}} \| \mathbf{x} \|_1 \quad \text{s.t.} \| \mathbf{r} - \mathbf{\Psi} \mathbf{x} \|_2 \le \epsilon, \tag{13}$$

which can be solved by the convex optimisation methods [29]. Moreover, by introducing an additional parameter λ , the following unconstrained optimisation problem can be also obtained [30]:

$$\min_{\boldsymbol{x}} \lambda \| \boldsymbol{x} \|_{1} + \frac{1}{2} \| \boldsymbol{r} - \boldsymbol{\Psi} \boldsymbol{x} \|_{2}.$$
(14)

By choosing the appropriate λ and ε , the same solutions in problem (13) and (14) can be achieved [31]. Furthermore, the ℓ_1 optimisation problem can be also expressed as a least absolute selection and shrinkage operator (LASSO) problem [32]

$$\min_{x} \| \boldsymbol{r} - \boldsymbol{\Psi} \boldsymbol{x} \|_{2} \quad \text{s.t.} \| \boldsymbol{x} \|_{1} \leq \epsilon', \tag{15}$$

where $\varepsilon' > 0$ and an appropriate ε' can guarantee the same solution with (13). Therefore, with appropriate parameters ε , λ and ε' , the same solutions can be achieved by the convex optimisation problems (13)–(15).

Since the ℓ_1 optimisation problems are convex, the Lagrange multipliers and the Karush–Kuhn–Tucker over interior-point method can be used. In addition, to further reduce the computational complexity, some efficient algorithms have been adopted, such as approximate message passing [33], fast adaptive shrinkage/thresholding algorithm (FASTA) [34], ℓ_1 -based spectral projection gradient [35]. Rather than the ℓ_1 optimisation problems, the greedy algorithms have also been proposed to solve the ℓ_0 optimisation problem. For example, matching pursuit (MP) [36], orthogonal MP (OMP) [37], stagewise OMP [38], regularised OMP [39], compressive sampling matching pursuit (CoSaMP) [40], have been proposed.

After establishing the CS-based radar system model in (11), we can adopt the typical CS-based methods to estimate the targets locations via reconstructing the sparse vector \mathbf{x} . Moreover, in this paper, to further improve the estimation performance, we propose a novel method to optimise the antennas placement, and more details about the proposed method are given in the following section.

4 Placement optimisation for MIMO antennas

Since the CS-based methods are adopted to estimate the targets locations, the localisation performance is determined by the CS reconstruction performance. Therefore, in this section, we are focusing on improving the reconstruction performance. In the CS

IET Radar Sonar Navig., 2017, Vol. 11 Iss. 2, pp. 285-293 © The Institution of Engineering and Technology 2016 theory, restricted isometry property (RIP) [29] guarantees the general reconstruction performance. To reconstruct a z-sparse signal x, the dictionary matrix Ψ must satisfy the following RIP condition for all the z-sparse signals x

$$(1 - \delta_z) \| \mathbf{x} \|_2^2 \le \| \mathbf{\Psi} \mathbf{x} \|_2^2 \le (1 + \delta_z) \| \mathbf{x} \|_2^2,$$
(16)

where δ_z is an isometry constant and not very closed to one. However, it is not practical to estimate the RIP of a dictionary matrix Ψ with large dimension. Therefore, the mutual coherence as an alternative of RIP is proposed [29]

$$\mu(\boldsymbol{\Psi}) \triangleq \max_{a \neq b} \frac{\left| \boldsymbol{\psi}_{a}^{\mathrm{H}} \boldsymbol{\psi}_{b} \right|}{\left\| \boldsymbol{\psi}_{a} \right\|_{2} \left\| \boldsymbol{\psi}_{b} \right\|_{2}},\tag{17}$$

where $\varepsilon_{;a}$ $(0 \le a \le PQ - 1)$ and $\varepsilon_{;b}$ $(0 \le b \le PQ - 1)$ denote the *a*th and *b*th columns of Ψ , respectively. In addition, with the mutual coherence $\mu(\Psi)$, the upper bound of RIP can be obtained as $\delta_z \le (z - 1)\mu(\Psi)$. Therefore, minimising the mutual coherence $\mu(\Psi)$ can further improve the reconstruction performance of the CS-based methods [14, 19]. In this paper, we propose a novel method to optimise the antennas locations of MIMO radar system and minimise the mutual coherence $\mu(\Psi)$.

With the definition of $\varepsilon_{;a}$ in (10), we have

$$\|\boldsymbol{\psi}_{a}\|_{2} = \sqrt{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left| e^{-j2\pi f_{c} \tau_{m,n,a}} \right|^{2}} = \sqrt{MN}, \qquad (18)$$

$$\| \boldsymbol{\psi}_{b} \|_{2} = \sqrt{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left| e^{-j2\pi f_{c} \tau_{m,n,b}} \right|^{2}} = \sqrt{MN},$$
(19)

so $\|\boldsymbol{\psi}_a\|_2 \|\boldsymbol{\psi}_b\|_2 = MN$. The numerator of (17) can be simplified as

$$\begin{aligned} \left| \boldsymbol{\psi}_{a}^{\mathrm{H}} \boldsymbol{\psi}_{b} \right| &= \left| \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \mathrm{e}^{\mathrm{j} 2 \pi f_{c} \boldsymbol{\tau}_{m,n,a}} \mathrm{e}^{-\mathrm{j} 2 \pi f_{c} \boldsymbol{\tau}_{m,n,b}} \right| \\ &= \left| \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} B_{a,b}(\boldsymbol{p}_{T,m}) B_{a,b}(\boldsymbol{p}_{R,n}) \right|, \end{aligned}$$
(20)

where we define

$$B_{a,b}(\boldsymbol{p}_{T,m}) \triangleq e^{j2\pi \frac{f_c}{c} \left(\| \boldsymbol{p}_{T,m} - \boldsymbol{t}_a \|_2 - \| \boldsymbol{p}_{T,m} - \boldsymbol{t}_b \|_2 \right)},$$
(21)

$$B_{a,b}(\boldsymbol{p}_{R,n}) \triangleq e^{j2\pi \frac{f_c}{c} \left(\| \boldsymbol{p}_{R,n} - \boldsymbol{t}_a \|_2 - \| \boldsymbol{p}_{R,n} - \boldsymbol{t}_b \|_2 \right)}.$$
 (22)

We define the sum of $B_{a,b}(\boldsymbol{p}_{T,m})$ and $B_{a,b}(\boldsymbol{p}_{R,n})$, respectively, as

$$A_{T,a,b} e^{j\beta_{T,a,b}} \triangleq \sum_{m=0}^{M-1} \frac{1}{M} B_{a,b}(\boldsymbol{p}_{T,m}), \quad 0 \le A_{T,a,b} \le 1, \quad (23)$$

$$A_{R,a,b} e^{j\beta_{R,a,b}} \triangleq \sum_{n=0}^{N-1} \frac{1}{N} B_{a,b}(\boldsymbol{p}_{R,n}), \quad 0 \le A_{R,a,b} \le 1,$$
(24)

where $A_{T,a,b}$ and $A_{R,a,b}$ denote the amplitudes, and $\beta_{T,a,b}$ and $\beta_{R,a,b}$ denote the phases. Then, we can obtain

$$\frac{1}{MN} |\boldsymbol{\psi}_{a}^{\mathrm{H}} \boldsymbol{\psi}_{b}| = \left\| \frac{1}{M} \sum_{m=0}^{M-1} B_{a,b}(\boldsymbol{p}_{T,m}) \right\| \left[\frac{1}{N} \sum_{n=0}^{N-1} B_{a,b}(\boldsymbol{p}_{R,n}) \right] \right\|$$

$$= \left| A_{T,a,b} e^{\mathrm{j}\beta_{T,a,b}} A_{R,a,b} e^{\mathrm{j}\beta_{R,a,b}} \right| = A_{T,a,b} A_{R,a,b} \,.$$
(25)

Therefore, the mutual coherence in (17) can be simplified as

Algorithm 1

1: *Input:* number of TXs M, number of RXs N, radar area $(0, x_0) \times (0, y_0)$. 2: Initialization: TX locations $p_{T,m} = 0_2$ $(1 \le m \le M)$, RX locations $p_{R,n} = 0_2$ $(1 \le n \le N)$, m = 1, n = 1, and l = 1.3: while $l \le \max\{M - 1, N - 1\}$ do if $m \leq M$ then 4: Discretize $(0, x_0) \times (0, y_0)$ into U positions $p_{T,m+1,u} \in \mathbb{R}^2$ $(0 \le u \le U - 1)$. 5: $\forall \boldsymbol{p}_{T,m+1,u}$, generate $\boldsymbol{\Psi}_{m+1,n,u}$ ($0 \le u \le U-1$) from (9). $\forall \boldsymbol{\Psi}_{m+1,n,u}$, calculate $\mu(\boldsymbol{\Psi}_{m+1,n,u})$ ($0 \le u \le U-1$) from (26). 6: 7: $u^* = \arg\min \mu (\Psi_{m+1,n,u}).$ 8: $p_{T,m+1} = p_{T,m+1,u^*}$ 9: 10: $m \leftarrow m + 1.$ end if 11: if $n \leq N$ then 12: Discretize $(0, x_0) \times (0, y_0)$ into V positions $\boldsymbol{p}_{R, n+1, v} \in \mathbb{R}^2$ $(0 \le v \le V - 1)$. 13: $\forall \boldsymbol{p}_{R,n+1,u}, \text{ generate } \boldsymbol{\Psi}_{m,n+1,u} \ (0 \leq v \leq V-1) \text{ from (9).} \\ \forall \boldsymbol{\Psi}_{m,n+1,v}, \text{ calculate } \mu \left(\boldsymbol{\Psi}_{m,n+1,v} \right) \ (0 \leq v \leq V-1) \text{ from (26).}$ 14: 15: $v^* = \arg\min\,\mu\,(\Psi_{m,n+1,u}).$ 16: $p_{R,n+1} \stackrel{0 \leq v \leq V-1}{=} p_{R,n+1,v^*}.$ 17: $n \leftarrow n+1.$ 18: end if 19: $l \leftarrow l+1.$ 20: 21: end while 22: Output: optimized TX locations $p_{T,m}$ ($0 \le m \le M-1$), optimized RX locations $p_{T,n}$ $(0 \le n \le N - 1).$

Fig. 3 Algorithm 1: The optimisation of antenna placement

$$\mu(\boldsymbol{\Psi}) \triangleq \frac{1}{MN} \max_{a \neq b} \left| \boldsymbol{\psi}_{a}^{\mathrm{H}} \boldsymbol{\psi}_{b} \right| = \max_{a \neq b} A_{T,a,b} A_{R,a,b} \,. \tag{26}$$

To minimise the mutual coherence $\mu(\Psi)$, we propose a method to add the TX and RX antennas iteratively, which is described in Algorithm 1 (see Fig. 3). When the number of TXs and RXs are,

respectively, *m* and *n*, the corresponding mutual corresponding $\mu(\Psi_{m,n})$ can be obtained. Then, we add one more TX, and the mutual coherence $\mu(\Psi_{m+1,n})$, the upper and lower bounds can be, respectively, obtained as (see (27) and (28)) (see (28) and (29)) (see (29))

$$\mu(\Psi_{m+1,n}) \triangleq \frac{1}{(m+1)n} \max_{a \neq b} \left| \sum_{n_1=0}^{n-1} \sum_{m_1=0}^{m} B_{a,b}(\boldsymbol{p}_{T,m_1}) B_{a,b}(\boldsymbol{p}_{R,n_1}) \right|$$

$$= \frac{1}{(m+1)n} \max_{a \neq b} \left| B_{a,b}(\boldsymbol{p}_{T,m}) \sum_{n_1=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_1}) + \sum_{n_1=0}^{n-1} \sum_{m_1=0}^{m-1} B_{a,b}(\boldsymbol{p}_{R,n_1}) B_{a,b}(\boldsymbol{p}_{T,m_1}) \right|,$$
(27)

$$\mu(\Psi_{m+1,n}) \leq \frac{1}{(m+1)n} \max_{a \neq b} \left\{ \left| B_{a,b}(\boldsymbol{p}_{T,m}) \sum_{n_1=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_1}) \right| + \left| \sum_{n_1=0}^{n-1} \sum_{m_1=0}^{m-1} B_{a,b}(\boldsymbol{p}_{R,n_1}) B_{a,b}(\boldsymbol{p}_{T,m_1}) \right| \right\}$$

$$\leq \frac{1}{(m+1)n} \max_{a \neq b} \left\{ \left| B_{a,b}(\boldsymbol{p}_{T,m}) \sum_{n_1=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_1}) \right| \right\} + \frac{m}{m+1} \mu(\Psi_{m,n})$$

$$\leq \frac{m\mu(\Psi_{m,n}) + 1}{m+1},$$
(28)

$$\mu(\Psi_{m+1,n}) \geq \frac{1}{(m+1)n} \max_{a \neq b} \left\{ \left\| B_{a,b}(\boldsymbol{p}_{T,m}) \sum_{n_{1}=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_{1}}) \right| - \left| \sum_{n_{1}=0}^{n-1} \sum_{m_{1}=0}^{m-1} B_{a,b}(\boldsymbol{p}_{R,n_{1}}) B_{a,b}(\boldsymbol{p}_{T,m_{1}}) \right| \right\| \right\} \\
= \frac{1}{(m+1)n} \max \left\{ \max_{a \neq b} \left\{ \left| B_{a,b}(\boldsymbol{p}_{T,m}) \sum_{n_{1}=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_{1}}) \right| - \left| \sum_{n_{1}=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_{1}}) \sum_{m_{1}=0}^{m-1} B_{a,b}(\boldsymbol{p}_{T,m_{1}}) \right| \right\} \right\} \\
\times \max_{a \neq b} \left\{ \left| \sum_{n_{1}=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_{1}}) \sum_{m_{1}=0}^{m-1} B_{a,b}(\boldsymbol{p}_{T,m_{1}}) \right| - \left| B_{a,b}(\boldsymbol{p}_{T,m}) \sum_{n_{1}=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_{1}}) \right| \right\} \right\} \\
\geq \max \left\{ \max_{a \neq b} \left\{ \frac{1}{(m+1)n} \left| B_{a,b}(\boldsymbol{p}_{T,m}) \sum_{n_{1}=0}^{n-1} B_{a,b}(\boldsymbol{p}_{R,n_{1}}) \right| - \frac{m}{m+1} \right\}, \frac{m\mu(\Psi_{m,n}) - 1}{m+1} \right\} \\
\geq \frac{m\mu(\Psi_{m,n}) - 1}{m+1}.$$
(29)

As shown in (28) and (29), after adding one more TX, the mutual coherence has the following lower and upper bounds:

$$\mu(\Psi_{m+1,n}) \in \left[\frac{m\mu(\Psi_{m,n}) - 1}{m+1}, \frac{m\mu(\Psi_{m,n}) + 1}{m+1}\right].$$
 (30)

Using the same method, after adding one more RX, the lower and upper bounds of mutual coherence can be obtained

$$\mu(\Psi_{m,n+1}) \in \left[\frac{n\mu(\Psi_{m,n}) - 1}{n+1}, \frac{n\mu(\Psi_{m,n}) + 1}{n+1}\right].$$
 (31)

Since we also have

$$\mu(\Psi_{m,n}) \in \left[\frac{m\mu(\Psi_{m,n}) - 1}{m+1}, \frac{m\mu(\Psi_{m,n}) + 1}{m+1}\right],$$
(32)

$$\mu(\Psi_{m,n}) \in \left[\frac{n\mu(\Psi_{m,n}) - 1}{n+1}, \frac{n\mu(\Psi_{m,n}) + 1}{n+1}\right],$$
(33)

We cannot guarantee that the mutual coherence is decreasing with adding one more TX and RX antennas randomly. Therefore, at Steps 8 and 16 of Algorithm 1 (Fig. 3), we choose the position for the additional TX or RX antenna via minimising the corresponding mutual coherence.

5 Statistical analysis

To describe the performance of Algorithm 1 (Fig. 3) in decreasing the mutual coherence, the distribution of mutual coherence with random antennas placement will be analysed in this section. Given the grid positions t_a and t_b , we define the following variables:

$$x_{T,a,b,m} \triangleq \| \boldsymbol{p}_{T,m} - \boldsymbol{t}_a \|_2 - \| \boldsymbol{p}_{T,m} - \boldsymbol{t}_b \|_2,$$
(34)

$$x_{R,a,b,n} \triangleq \| \boldsymbol{p}_{R,n} - \boldsymbol{t}_{a} \|_{2} - \| \boldsymbol{p}_{R,n} - \boldsymbol{t}_{b} \|_{2}.$$
(35)

When *M* TXs and *N* RXs are chosen randomly in the area $(0, x_0) \times (0, y_0)$ with independent and identical distributions, $x_{T,a,b,m}$ and $x_{R,a,b,n}$ also have independent and identical distributions for different parameters *m* and *n*.

Define the probability density functions (PDFs) of $x_{T,a,b,m}$ and $x_{R,a,b,n}$ as $f_{x_{T,a,b}}(x_{T,a,b,m})$ and $f_{x_{R,a,b}}(x_{R,a,b,n})$, respectively. Then, according to [41], the PDFs of amplitudes $A_{T,a,b}$ and $A_{R,a,b}$ in (23) and (24) can be expressed as

$$f_{A_{T,a,b}}(A_{T,a,b}) = \frac{A_{T,a,b}}{2\pi\sqrt{s_{T_1}s_{T_2}}} \int_0^{2\pi} \exp\left[-\frac{(A_{T,a,b}\cos\theta - \gamma_{T_1})^2}{2s_{T_1}} - \frac{(A_{T,a,b}\sin\theta - \gamma_{T_2})^2}{2s_{T_2}}\right] d\theta,$$
(36)

$$f_{A_{R,a,b}}(A_{R,a,b}) = \frac{A_{R,a,b}}{2\pi\sqrt{s_{R1}s_{R2}}} \int_{0}^{2\pi} \exp\left[-\frac{\left(A_{R,a,b}\cos\theta - \gamma_{R1}\right)^{2}}{2s_{R1}} - \left[\frac{\left(A_{R,a,b}\sin\theta - \gamma_{R2}\right)^{2}}{2s_{R2}}\right] d\theta, \quad 0 \le A_{R,a,b} \le 1,$$
(37)

where

$$\begin{split} \gamma_{T1} &= \int f_{x_{T,a,b}}(x) \cos\left(\frac{1}{c} 2\pi f_c x\right) \mathrm{d}x, \\ \gamma_{T2} &= \int f_{x_{T,a,b}}(x) \sin\left(\frac{1}{c} 2\pi f_c x\right) \mathrm{d}x, \\ \gamma_{R1} &= \int f_{x_{R,a,b}}(x) \cos\left(\frac{1}{c} 2\pi f_c x\right) \mathrm{d}x, \\ \gamma_{R2} &= \int f_{x_{R,a,b}}(x) \sin\left(\frac{1}{c} 2\pi f_c x\right) \mathrm{d}x, \end{split}$$

and

$$\begin{split} s_{T_1} &= \frac{1}{M} \int f_{x_{T,a,b}}(x) \cos^2 \left(\frac{1}{c} 2\pi f_c x\right) dx - \gamma_{T_1}^2, \\ s_{T_2} &= \frac{1}{M} \int f_{x_{T,a,b}}(x) \sin^2 \left(\frac{1}{c} 2\pi f_c x\right) dx - \gamma_{T_2}^2, \\ s_{R_1} &= \frac{1}{N} \int f_{x_{R,a,b}}(x) \cos^2 \left(\frac{1}{c} 2\pi f_c x\right) dx - \gamma_{R_1}^2, \\ s_{R_2} &= \frac{1}{N} \int f_{x_{R,a,b}}(x) \sin^2 \left(\frac{1}{c} 2\pi f_c x\right) dx - \gamma_{R_2}^2. \end{split}$$

With the PDFs of $A_{T,a,b}$ and $A_{R,a,b}$ in (36) and (37), the cumulative distribution function (CDF) of random variable $X_{a,b} \triangleq A_{T,a,b}A_{R,a,b}$ can be obtained (see (38)) where $P(\cdot)$ is the probability function. Therefore, the CDF of mutual coherence defined in (26) can be obtained as

$$F_{\mu}(\mu_{0}) = P(\mu(\Psi) \le \mu_{0})$$

= $P(\max_{a \ne b} X_{a,b} \le \mu_{0})$
 $\simeq \prod_{a=0}^{PQ-2} \prod_{b=a+1}^{PQ-1} F_{X_{a,b}}(\mu_{0}),$ (39)

where the approximation is based on the assumption that $X_{a,b}$ is independent for different *a* and *b*. Then, with the CDF of mutual coherence $\mu(\Psi)$, we can measure the performance of Algorithm 1 (Fig. 3) via the probability that the mutual coherence of random TXs and RXs locations is less than the one with optimised locations. In addition, in the following subsection, the asymptotic distribution of $\mu(\Psi)$ with large *M* and *N* will also be given.

5.1 Asymptotical behaviour of the mutual coherence

Since the variables $B_{a,b}(\mathbf{p}_{T,m})$ and $B_{a,b}(\mathbf{p}_{T,m})$ defined in (21) and (22) follow the independent and identical distributions, then, we define the following variables:

$$X_{T,a,b} \triangleq \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \left[B_{a,b}(\boldsymbol{p}_{T,m}) - \mu_{T,a,b} \right], \tag{40}$$

$$X_{R,a,b} \triangleq \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left[B_{a,b}(\boldsymbol{p}_{R,n}) - \mu_{R,a,b} \right],$$
(41)

where

$$\mu_{T,a,b} = \mathbb{E}_{\boldsymbol{p}_{T,m}} \{ B_{a,b}(\boldsymbol{p}_{T,m}) \}$$

= $\int e^{j2\pi(f_c/c)x_{T,a,b}} f(x_{T,a,b}) dx_{T,a,b} = \zeta_{x_{T,a,b}} \left(2\pi \left(\frac{f_c}{c} \right) \right),$ (42)

$$F_{X_{a,b}}(\mu_0) = P(X_{a,b} \le \mu_0)$$

$$= \int_0^1 \int_0^{(\mu_0/A_{T,a,b})} f_{A_{T,a,b}}(A_{T,a,b}) f_{A_{R,a,b}}(A_{R,a,b}) \, \mathrm{d}A_{R,a,b} \, \mathrm{d}A_{T,a,b},$$
(38)

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$$\mu_{R,a,b} = \mathbb{E}_{\boldsymbol{p}_{R,m}} \{ B_{a,b}(\boldsymbol{p}_{R,m}) \}$$

= $\int e^{j2\pi \frac{f_c}{c} x_{R,a,b}} f(x_{R,a,b}) dx_{R,a,b} = \zeta_{x_{R,a,b}} \left(2\pi \frac{f_c}{c} \right)^{.}$ (43)

 $\zeta_{x_{T,a,b}}(2\pi(f_c/c))$ and $\zeta_{x_{R,a,b}}(2\pi(f_c/c))$ are the characteristic functions of $f(x_{T,a,b})$ and $f(x_{R,a,b})$, respectively.

Therefore, according to Lyapunov's central limit theorem [42], both $X_{R,a,b}$ and $X_{T,a,b}$ follow the asymptotic complex Gaussian distribution with large enough M and N, i.e.

$$\begin{split} X_{T,a,b} &\sim \mathscr{CN}\big(0, \Gamma_{T,a,b}, C_{T,a,b}\big), X_{R,a,b} \sim \mathscr{CN}\big(0, \Gamma_{R,a,b}, C_{R,a,b} \left(44\right)\big), \end{split}$$

where

$$\begin{split} \mathbf{\Gamma}_{T,a,b} &\triangleq \mathbb{E}_{\mathbf{p}_{T,n}} \left\{ B_{a,b}(\mathbf{p}_{T,n}) B_{a,b}^{\mathrm{H}}(\mathbf{p}_{T,n}) \right\} = 1, \\ \mathbf{\Gamma}_{R,a,b} &\triangleq \mathbb{E}_{\mathbf{p}_{R,n}} \left\{ B_{a,b}(\mathbf{p}_{R,n}) B_{a,b}^{\mathrm{H}}(\mathbf{p}_{R,n}) \right\} = 1, \\ \mathbf{C}_{T,a,b} &\triangleq \mathbb{E}_{\mathbf{p}_{T,n}} \left\{ B_{a,b}(\mathbf{p}_{T,n}) B_{a,b}(\mathbf{p}_{T,n}) \right\} = \zeta_{x_{T,a,b}} \left(4\pi \frac{f_{\mathrm{c}}}{c} \right), \\ \mathbf{C}_{R,a,b} &\triangleq \mathbb{E}_{\mathbf{p}_{R,n}} \left\{ B_{a,b}(\mathbf{p}_{R,n}) B_{a,b}(\mathbf{p}_{R,n}) \right\} = \zeta_{x_{R,a,b}} \left(4\pi \frac{f_{\mathrm{c}}}{c} \right), \end{split}$$

and $\mathscr{CN}(0, \Gamma_{T,a,b}, C_{T,a,b})$ denotes the zero-mean complex Gaussian distribution with covariance matrix being $\Gamma_{T,a,b}$ and relation matrix being $C_{T,a,b}$.

Since we have the following linear relationship:

$$A_{T,a,b} e^{j\beta_{T,a,b}} = \frac{1}{\sqrt{M}} X_{T,a,b} + \mu_{T,a,b},$$
(45)

$$A_{R,a,b} e^{j\beta_{R,a,b}} = \frac{1}{\sqrt{N}} X_{R,a,b} + \mu_{R,a,b},$$
(46)

both $A_{T,a,b} e^{j\beta T,a,b}$ and $A_{R,a,b} e^{j\beta R,a,b}$ also follow the joint complex Gaussian distributions, i.e.

$$A_{T,a,b} e^{j\beta_{T,a,b}} \sim \mathscr{CN}\left(\mu_{T,a,b}, \frac{1}{M}, \frac{C_{T,a}}{M}\right),$$

$$A_{R,a,b} e^{j\beta_{R,a,b}} \sim \mathscr{CN}\left(\mu_{R,a,b}, \frac{1}{N}, \frac{C_{R,a,b}}{N}\right).$$
(47)

With the distribution of $A_{T,a,b} e^{j\beta T,a,b}$ and $A_{R,a,b} e^{j\beta R,a,b}$, the distribution of $X_{a,b} = A_{T,a,b}A_{R,a,b}$ can be obtained. However, it is difficult to obtain the explicit distribution expression of $X_{a,b}$. Therefore, in this paper, we will give an approximation expressed under the following conditions:

$$\mu_{T,a,b} = 0, \quad \mu_{R,a,b} = 0, \quad C_{T,a,b} = 0, \quad C_{R,a,b} = 0,$$

which can be guaranteed when the following condition can be satisfied:

$$2\| t_a - t_b \|_{\gamma} f_c \gg 1.$$
(48)

Then, we have

$$A_{T,a,b} e^{j\beta_{T,a,b}} \sim \mathscr{CN}\left(0, \frac{1}{M}, 0\right), \quad A_{R,a,b} e^{j\beta_{R,a,b}} \sim \mathscr{CN}\left(0, \frac{1}{N}, 0\right),$$

$$(49)$$

and

$$\begin{cases} \operatorname{Re}\left\{A_{T,a,b} \,\mathrm{e}^{\mathrm{j}\beta_{T,a,b}}\right\} \\ \operatorname{Im}\left\{A_{T,a,b} \,\mathrm{e}^{\mathrm{j}\beta_{T,a,b}}\right\} \end{cases} \sim \mathscr{CN} \left[\begin{bmatrix} 0\\0 \end{bmatrix}, \frac{1}{2} \begin{bmatrix} \frac{1}{M}, & 0\\0, & \frac{1}{M} \end{bmatrix} \right], \tag{50}$$

$$\begin{pmatrix} \operatorname{Re}\left\{A_{R,a,b} e^{j\beta_{R,a,b}}\right\} \\ \operatorname{Im}\left\{A_{R,a,b} e^{j\beta_{R,a,b}}\right\} \end{pmatrix} \sim \mathscr{CN} \begin{pmatrix} 0\\0 \end{pmatrix}, \frac{1}{2} \begin{bmatrix} \frac{1}{N}, & 0\\0, & \frac{1}{N} \end{bmatrix} \end{pmatrix}.$$
(51)

Therefore, the amplitudes $A_{T,a,b}$ and $A_{R,a,b}$ follow the Rayleigh distribution:

$$A_{T,a,b} \sim \text{Rayleigh}\left(\frac{1}{\sqrt{2M}}\right), \quad A_{R,a,b} \sim \text{Rayleigh}\left(\frac{1}{\sqrt{2N}}\right).$$
 (52)

 $X_{a,b}$ is a product between two independent variables with Rayleigh distribution, and the CDF of $X_{a,b}$ can be expressed according to [43]

$$F_{X_{a,b}}(x) = 1 - \sqrt{2MN} x K_1(\sqrt{2MN}x), \quad x \ge 0,$$
(53)

where $K_1(\cdot)$ denotes the modified Bessel functions of the second kind.

Finally, with $M \gg 0$ and $N \gg 0$, the CDF of mutual coherence $\mu(\Psi)$ can be obtained

$$F'_{\mu}(\mu_{0}) = P(\mu(\Psi) \le \mu_{0})$$

= $P(\max_{a \ne b} X_{a,b} \le \mu_{0})$
 $\simeq [1 - \mu'_{0}K_{1}(\mu'_{0})]^{(1/2)PQ(PQ-1)},$ (54)

where $\mu'_0 \triangleq 2\sqrt{MN}\mu_0$.

In Section 6, we will compare the CDF of $\mu(\Psi)$ described by both (39) and (54), and illustrate the performance of Algorithm 1 (Fig. 3) in decreasing the mutual coherence via optimising the antennas locations.

6 Simulation results

In this section, we simulate the location performance for multiple targets, where the antennas locations are optimised, and the CS-based methods including OMP, FASTA-LASSO and CoSaMP are adopted. During the simulations, the range resolution is $\Delta = 1$ m, and we discretise the target area into an $10\Delta \times 10\Delta$ area, i.e. P = 10 and Q = 10. The carrier frequency is $f_c = 1$ GHz and the speed of electromagnetic wave is $c = 10^8$ m/s. The number of TXs *M* is chosen from 4 to 10, and the number of RXs *N* is also chosen from 4 to 10.

6.1 Analysis of Algorithm 1 (Fig. 3)

First, the mutual coherence with increasing the antennas number in Algorithm 1 (Fig. 3) is shown in Fig. 4, where the TX and RX antennas are added alternatively. As shown in this figure, increasing the antennas number can decrease the mutual coherence of dictionary matrix in (9). In addition, the upper and lower bounds of adding one more TX or RX antenna is also illustrated, where the upper and lower bounders can be obtained in (28) and (29), respectively. The obtained mutual coherence for different antennas is between these upper and lower bounds, and these bounds also decrease with increasing the antennas number. Therefore, increasing the antennas number in Algorithm 1 (Fig. 3) is an effective method to decrease the mutual coherence of dictionary matrix.

In Fig. 5, the CDF of mutual coherence is given, where the empirical results are obtained by the 10^5 times of simulations, the theoretical results are obtained by (39), and the asymptotically theoretical results are obtained by (54) with the TX number $M \gg 0$ and the RX number $N \gg 0$. As shown in this figure, the CDFs of

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Fig. 4 Mutual coherence with increasing the number of antennas in Algorithm 1 (Fig. 3)



Fig. 5 *CDF of mutual coherence* $\mu(\Psi)$



Fig. 6 *Comparison between the mutual coherence optimised by Algorithm 1 (Fig. 3) and the one with random antennas placement*

mutual coherence with antennas numbers being $M = N = \{4, 6, 8, 10\}$ are given, and the probability $P(\mu \le \mu_0)$ is increasing with increasing the antennas number, which means that we have more probability to obtain a smaller mutual coherence with more TX and RX antennas. In addition, to obtain the CDF (39), we assume that the random variables $X_{a,b}$ in (38) are independent for different pairs of *a* and *b*. Therefore, there are some differences between the empirical CDF and the theoretical CDF (39), but these differences are decreasing with the increasing number of antennas. When M =N = 10, the differences are too small, and the theoretical CDF (39) has a good description of the simulation results. Furthermore, we also propose (54) to describe the CDF of mutual coherence when $M \gg 0$ and $N \gg 0$. Since the range resolution $|| \mathbf{t}_a - \mathbf{t}_b ||_2 \gg 1/2f_c$, the assumption in (48) can be satisfied easily. When the antennas number is less, e.g. M=N=4, the asymptotically theoretical (54) cannot describe the simulation results well, but when $M=N\geq 8$, the asymptotically theoretical (54) can describe the simulated CDF well. Therefore, when $M=N\leq 8$, the theoretical CDF in (39) is preferred to describe the CDF of mutual coherence, and when M=N>8, both the theoretical CDF in (39) and the asymptotic one in (54) can be used.

With the theoretical and simulated CDF, we can obtain the probability that the mutual coherence of optimising the TX and RX locations based on Algorithm 1 (Fig. 3) is less than a given one with random TX and RX locations. As shown in Fig. 6, the mutual coherences with optimised TX and RX locations and different antennas numbers are given. In addition, when the probability that the mutual coherence with random TX and RX locations is smaller than μ_0 is 10⁻³, the mutual coherences μ_0 are also shown. The mutual coherences μ_0 are obtained by the empirical CDF, the theoretical CDF (39) and the asymptotically theoretical CDF (54), respectively. As shown in this figure, the mutual coherences obtained by optimising the TX and RX locations are smaller than μ_0 , which means that the probability that smaller mutual coherences can be achieved by random TX and RX locations is less than 10^{-3} . Therefore, Algorithm 1 (Fig. 3) is effective in obtaining the smaller mutual coherences via optimising TX and RX locations.

6.2 Multiple targets localisations performance

In this subsection, the performance of target localisations is given, where the localisation performance is measured by the probability of reconstructing the support set of sparse vector \mathbf{x} . The target number is K = 5, and the number of TX antennas is equal to that of RX antennas, i.e. $M = N \in \{4, 5, ..., 10\}$. The received waveforms are interfered by AWGN, and the signal-to-noise ratio (SNR) is SNR $\in \{3, 6, 9\}$ dB. The target localisation performance with both optimised and random antennas locations is given, where the localisation performance is realised by averaging the 10^6 times of reconstructing sparse vector, and 10^6 times of antennas positions are randomly chosen to obtain the non-optimised localisation performance.

We choose three typical CS reconstruction methods including FASTA-LASSO, OMP and CoSaMP, where the support recovery performance is shown in Figs. 7*a*, 8*a* and 9*a*, respectively. In these three methods, the support recovery performance is all improved by optimised the TX and RX positions, especially, in the higher SNR situations (SNR ≥ 6 dB). Moreover, with the increasing the antennas number, the support recovery performance is also improved.

In Figs. 7b, 8b and 9b, the localisation performance of FASTA-LASSO, OMP and CoSaMP is given, respectively. The localisation performance is measured by the mean squared error. As shown in these figures, the localisation performance is consistent with the sparse reconstruction performance. Therefore, the placement optimisation can improve both the sparse reconstruction and localisation performance. In addition, the localisation performance can be more significantly improved when the MIMO radar system equips more antennas. Therefore, by optimising the TX and RX positions with Algorithm 1 (Fig. 3), the target localisation performance can be improved, especially at high SNR or with more antennas.

Furthermore, as shown in Fig. 10, we also compared the proposed method of the placement optimisation with an existing method proposed in [44], where the radar placement is optimised according to the Neyman–Pearson detector. In this comparison, the FASTA-LASSO algorithm is adopted to reconstruct the sparse vector. Since the placement optimisation is not to maximise the sparse reconstruction performance in the existing method, the proposed method achieves better performance in reconstructing the sparse vector. Therefore, the proposed method is more suitable for the CS-based target localisation.



Fig. 7 FASTA-LASSO algorithm (a) Support recovery performance, (b) Localisation performance



Fig. 8 OMP algorithm

(a) Support recovery performance, (b) Localisation performance







Fig. 9 CoSaMP algorithm

(a) Support recovery performance, (b) Localisation performance

7 Conclusions

We have addressed the localisation problem for multiple targets in the distributed MIMO radar system, where the localisation problem has been established as a sparse reconstruction problem based on the CS theory. To improve the target localisation performance, which corresponds to the sparse reconstruction performance in the CS problem, the placement of both TXs and RXs are optimised by the proposed iteration algorithm, and the mutual coherence of dictionary matrix is minimised. Moreover, both the theoretical and asymptotic distributions of mutual coherence with random

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Fig. 10 Support recovery performance compared with an existing method

antennas placement has also been provided, which can be adopted to measure the performance improvement in decreasing the mutual coherence by optimising the TXs and RXs placements. Then, simulation results have also been given, where typical CS-based methods are adopted and the performance of targets localisation are significantly improved by optimising the antennas placement. Further work will focus on the dynamic optimisation for the moving radar platforms.

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