Channel Estimation for Reconfigurable Intelligent Surface Aided Massive MIMO System

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***Abstract*—The uplink channel estimation for the reconfigurable intelligent surface (RIS) aided massive multiple-input multiple-output (MIMO) system is investigated. By changing the phase matrix of the RIS, multiple channel measurements for the channel estimation are obtained. During the initial channel estimation that represents the initial state of the system, an iterative algorithm is proposed to estimate the channel matrix between the base station (BS) and the RIS, where the initial value of the iterative algorithm is well selected. After that, the channel matrix between the RIS and the user equipment (UE) is estimated. During the regular channel estimation where the channel matrix between the BS and the RIS is supposed to be known, the channel matrix between the RIS and the UE and that between the BS and the UE are sequentially estimated with the consideration on the pilot overhead. Simulation results verify the effectiveness of the proposed channel estimation methods.**

***Index Terms*—Channel estimation, massive MIMO, reconfigurable intelligent surface (RIS).**

I. Introduction

With the fast development of artificial intelligence (AI), intelligent communications that incorporate AI into communications will be an inevitable trend for future wireless communications [1]–[4]. On the other hand, the intelligent environment is also on focus, where the reconfigurable intelligent surface (RIS) is developed to smartly control the wireless propagative environment [5]. Compared with traditional multiple-input multiple-output (MIMO) wireless system, the RIS aided MIMO system applies the RIS to reflect the signals with controllable phase shifts, which can achieve higher spectral efficiency and improve the signal coverage [6], [7]. The system model and architecture of the RIS aided MIMO system are similar to those of MIMO wireless relaying system. But different from the relay, the RIS can only passively reflect incident signals instead of positively transmitting signals. The RIS equipped with integrated electronic circuits can be readily fabricated using lithography and nano-printing methods, indicating that the RIS is much smaller and simpler than the relay. Most RIS is incapable of signal processing. Nevertheless, it has been shown that the RIS-based resource allocation methods are able to provide up to 300% higher energy efficiency in comparison with the regular multi-antenna amplify-and-forward relaying [8].

The passive feature of the RIS leads to the challenge of the channel estimation for the RIS aided massive MIMO system. In [9], the cascaded channel estimation is formulated as a combined sparse matrix factorization and matrix completion problem, where a two-stage algorithm is proposed. In [10], a practical transmission protocol is proposed to execute the channel estimation and reflection optimization successively for an RIS-enhanced orthogonal frequency division multiplexing (OFDM) system. However, the existing works do not make full use of the RIS smart controller to control phase shifts.

In this work, we investigate the uplink channel estimation for the RIS aided massive MIMO system. By changing the phase matrix of the RIS, we obtain multiple channel measurements for the channel estimation. During the initial channel estimation that represents the initial state of the system, we first propose an iterative algorithm to estimate the channel matrix between the base station (BS) and the RIS, where the initial value of the iterative algorithm is well selected. Then we estimate the channel matrix between the RIS and the user equipment (UE). During the regular channel estimation where the channel matrix between the BS and the RIS is supposed to be known, we sequentially estimate the channel matrix between the RIS and the UE and that between the BS and the UE, where the pilot overhead is considered.

We use the following notations in our paper. Symbols for vectors (lower case) and matrices (upper case) are in boldface. $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$ and $(\cdot)^\dagger$ denote the transpose, conjugate transpose (Hermitian), inverse and pseudo-inverse, respectively. We use $\boldsymbol{I}_n$ to represent the identity matrix of the size of n. The set of $p \times q$ complex-valued matrices is denoted by $\mathbb{C}^{p\times q}$. We use $\mathrm{vec}(\cdot)$ to denote the vectorization of the matrix, which means putting the column vectors of the matrix in sequence to form a long column vector. We use $\boldsymbol{A}(m,n)$ to denote the entry in the nth row and nth column of a matrix $\boldsymbol{A}$. $\|\boldsymbol{A}\|_F$ is used to denote the Frobenius norm of a matrix $\boldsymbol{A}$. $\lceil\cdot\rceil$ denotes the ceiling operation. $\mathcal{CN}(\mathbf{0},\boldsymbol{R})$ denotes the complex Gaussian distribution with zero mean and covariance matrix being $\boldsymbol{R}$. We use $\mathbb{E}[X]$ to denote the expectation of a random variable X and $\boldsymbol{A}\otimes\boldsymbol{B}$ to denote the kronecker product of two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.

II. System Model

As shown in Fig. 1, an RIS aided massive MIMO wireless communication system includes a BS, an RIS, a UE and an RIS smart controller. The RIS smart controller has high-speed wired connection with the RIS and the BS, enabling the BS to

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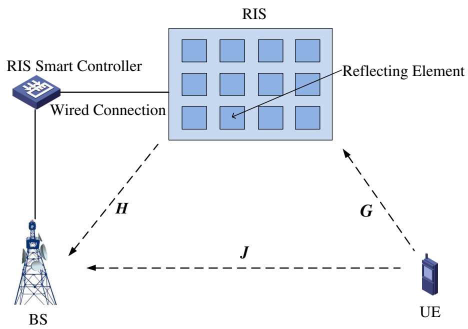

Fig. 1. Illustration of RIS aided MIMO wireless communication system.

control the RIS in real time. Note that the RIS is equipped with many reflecting elements, which can only passively reflect the incident signal instead of intentionally transmitting signal.

We consider the uplink transmission, where the UE transmits the signal to the BS. Denote the numbers of antennas at the BS, UE and RIS as N_{r}, N_{t} and N_{s}, respectively. Then the uplink channel between the UE and the RIS is denoted as an N_{s} by N_{t} matrix $\boldsymbol{G}$, where each entry of $\boldsymbol{G}$ independently obeys the same complex Gaussian distribution $\mathcal{CN}(0,\sigma_1^2)$ [11]. Similarly, the uplink channel between the RIS and the BS is denoted as an N_{r} by N_{s} matrix $\boldsymbol{H}$, where each entry of $\boldsymbol{H}$ independently obeys the same complex Gaussian distribution $\mathcal{CN}(0,\sigma_2^2)$. The uplink channel between the UE and the BS is denoted as an N_{r} by N_{t} matrix $\boldsymbol{J}$, where each entry of $\boldsymbol{J}$ independently obeys the same complex Gaussian distribution $\mathcal{CN}(0,\sigma_3^2)$. Note that the other channel models may also be fit for our work.

To ease the channel estimation of $\boldsymbol{J}$, we can first power off the RIS using the RIS smart controller. Then the channel estimation of $\boldsymbol{J}$ is simplified to be a peer-to-peer traditional MIMO channel estimation problem that can be tackled by the existing methods. Once $\boldsymbol{J}$ is estimated, we power on the RIS. When performing the channel estimation for the RIS aided MIMO wireless communication system, the received signal component from $\boldsymbol{J}$ can be treated as a known constant, which implies the impact of $\boldsymbol{J}$ can be completely removed. Therefore, in this work, we focus on the channel estimation of $\boldsymbol{G}$ and $\boldsymbol{H}$.

The RIS functioning as phase shifters on the incident signal, is typically modeled as a diagonal matrix, where each diagonal entry is independently controlled by a phase. We model the function of the RIS on the lth time of control for $l=0,1,\ldots,L-1$ as the following diagonal matrix

$$\boldsymbol{\Phi}_l=\begin{bmatrix} e^{-j\theta_1(l)} & 0 & \cdots & 0 \\ 0 & e^{-j\theta_2(l)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j\theta_{N_{\mathrm{s}}}(l)} \end{bmatrix} \tag{1}$$

where $\theta_1(l),\theta_2(l),\ldots,\theta_{N_{\mathrm{s}}}(l)$ are the N_{s} phases offering the degree of freedom to control the incident signal. Note that the circuit structure of the RIS smart controller is simple and cost-effective so that the change of the RIS phase shifts is easy and quick enough [5].

The received signal by the BS can be expressed as

$$\boldsymbol{Y}_l=\boldsymbol{H}\boldsymbol{\Phi}_l\boldsymbol{G}\boldsymbol{X}+\boldsymbol{N}_l \tag{2}$$

where $\boldsymbol{X}\in\mathbb{C}^{N_{\mathrm{t}}\times P}$ and $\boldsymbol{Y}\in\mathbb{C}^{N_{\mathrm{r}}\times P}$ are the transmitted pilot matrix by the UE and the received pilot matrix by the BS, respectively, and P represents the number of groups of pilots. $\boldsymbol{N}_l$ is a matrix made up of noise, where each entry of $\boldsymbol{N}_l$ independently obeys the complex Gaussian distribution $\mathcal{CN}(0,\sigma_{\mathrm{N}}^2)$. The objective of the channel estimation is essentially to obtain $\boldsymbol{G}$ and $\boldsymbol{H}$, given $\boldsymbol{X}$, $\boldsymbol{Y}_l$ and $\boldsymbol{\Phi}_l$.

III. Channel Estimation

The channel estimation for the RIS aided MIMO wireless communications is divided into two parts, including the initial channel estimation and regular channel estimation, which will be presented in the following two subsections. In the initial channel estimation, the channels between the BS and the RIS are unknown, which represents the initial state or some rare cases of the RIS aided MIMO wireless system. In the regular channel estimation, the channels between the BS and the RIS are supposed to be known, which represents most cases since the link between the BS and the RIS is normally better than that between the RIS and the UE.

A. Initial Channel Estimation

From L measurements of (2), we select one as the reference. Without loss of generality, we select $l=0$ as the reference, having

$$\boldsymbol{Y}_0=\boldsymbol{H}\boldsymbol{\Phi}_0\boldsymbol{G}\boldsymbol{X}+\boldsymbol{N}_0. \tag{3}$$

We assume $\boldsymbol{H}$ has full column rank and $N_{\mathrm{s}}\le N_{\mathrm{r}}$.[1] Then the pseudo-inverse of $\boldsymbol{H}$ can be expressed as

$$\boldsymbol{H}^{\dagger}=(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H, \tag{4}$$

with $\boldsymbol{H}^{\dagger}\boldsymbol{H}=\boldsymbol{I}_{N_{\mathrm{s}}}$. Note that $\boldsymbol{H}^{\dagger}$ is also unknown and needs to be estimated.

We multiply each side of (2) with $\boldsymbol{H}^{\dagger}$ for $l=1,2,\ldots,L-1$ with $L\ge 2$ and have

$$\boldsymbol{H}^{\dagger}\boldsymbol{Y}_l=\boldsymbol{\Phi}_l\boldsymbol{G}\boldsymbol{X}+\boldsymbol{H}^{\dagger}\boldsymbol{N}_l \tag{5}$$

Then we have

$$\boldsymbol{G}\boldsymbol{X}=\boldsymbol{\Phi}_l^{-1}\boldsymbol{H}^{\dagger}\boldsymbol{Y}_l-\boldsymbol{\Phi}_l^{-1}\boldsymbol{H}^{\dagger}\boldsymbol{N}_l. \tag{6}$$

By substituting (6) into (3), we have

$$\boldsymbol{Y}_0=\boldsymbol{H}\boldsymbol{\Phi}_0\boldsymbol{\Phi}_l^{-1}\boldsymbol{H}^{\dagger}\boldsymbol{Y}_l+\boldsymbol{N}_0-\boldsymbol{H}\boldsymbol{\Phi}_0\boldsymbol{\Phi}_l^{-1}\boldsymbol{H}^{\dagger}\boldsymbol{N}_l. \tag{7}$$

We define

$$\boldsymbol{F}_l\triangleq\boldsymbol{H}\boldsymbol{\Phi}_0\boldsymbol{\Phi}_l^{-1}\boldsymbol{H}^{\dagger},\ l=1,2,\ldots,L-1. \tag{8}$$

To estimate $\boldsymbol{H}$, we first estimate $\boldsymbol{F}_l$. Note that $\boldsymbol{N}_0-\boldsymbol{H}\boldsymbol{\Phi}_0\boldsymbol{\Phi}_l^{-1}\boldsymbol{H}^{\dagger}\boldsymbol{N}_l$ is a noise term. In order to achieve the

[1] If $N_{\mathrm{s}}>N_{\mathrm{r}}$, we can divided the RIS elements into several groups, where the BS sequentially power on one group and power off the other groups for the estimation.

estimation of $\boldsymbol{F}_l$, $\boldsymbol{Y}_l$ should have the right inverse matrix. In other words, we assume that

$$P \geq N_{\rm r}. \tag{9}$$

Then the least square (LS) estimation of $\boldsymbol{F}_l$ can be written as

$$\widehat{\boldsymbol{F}}_l = \boldsymbol{Y}_0 \boldsymbol{Y}_l^H (\boldsymbol{Y}_l \boldsymbol{Y}_l^H)^{-1},\ l = 1, 2, \ldots, L-1. \tag{10}$$

We now estimate $\boldsymbol{H}$ based on $\widehat{\boldsymbol{F}}_l, l = 1, 2, \ldots, L-1$. According to (8), we have

$$\widehat{\boldsymbol{F}}_l \boldsymbol{H} = \boldsymbol{H} \boldsymbol{\Phi}_0 \boldsymbol{\Phi}_l^{-1}. \tag{11}$$

Obviously, zero matrix is a solution of $\boldsymbol{H}$ to (11), which implies that (11) may have more than one solution. Since both $\boldsymbol{\Phi}_0$ and $\boldsymbol{\Phi}_l$ are full-rank diagonal matrix, we have

$$\boldsymbol{H} = \widehat{\boldsymbol{F}}_l \boldsymbol{H} \boldsymbol{\Phi}_l \boldsymbol{\Phi}_0^{-1}. \tag{12}$$

It is seen that both sides of (12) have $\boldsymbol{H}$. Therefore, we may choose an initial value for $\boldsymbol{H}$, denoted as $\boldsymbol{H}_0$, and then iteratively run

$$\boldsymbol{H}_{k+1} = \widehat{\boldsymbol{F}}_l \boldsymbol{H}_k \boldsymbol{\Phi}_l \boldsymbol{\Phi}_0^{-1} \tag{13}$$

until the stop condition

$$\|\boldsymbol{H}_{k+1} - \boldsymbol{H}_k\|_F < \epsilon \tag{14}$$

is satisfied, where k is the number of iteration and ϵ is a predefined threshold.

The selection of $\boldsymbol{H}_0$ is important, as it determines the convergence speed of the iteration. Sometimes it may even not converge, if $\boldsymbol{H}_0$ is not properly selected. Similar problem is investigated under the framework of MIMO wireless relaying [12], where the channel estimation is performed by using the signal amplification and exchange on the relay node.

To be more clear, we rewrite (11) as $N_{\rm r} N_{\rm s}$ equations, which are divided into $N_{\rm r}$ groups, with each group having $N_{\rm s}$ equations. The ith group is expressed in (15), where $i = 1, 2, \ldots, N_{\rm r}$. Define

$$\boldsymbol{h} = \mathrm{vec}(\boldsymbol{H}). \tag{16}$$

Then we have

$$\boldsymbol{Q}_l \boldsymbol{h} = \boldsymbol{0} \tag{17}$$

where $\boldsymbol{Q}_l$ is expressed in (18). In fact, the mth equation in the ith group is essentially the $\big((i-1)N_{\rm s} + m\big)$th equation of (17), for $i = 1, 2, \ldots, N_{\rm r}$ and $m = 1, 2, \ldots, N_{\rm s}$.

Based on (18), we can rewrite $\boldsymbol{Q}_l$ in a compact form as

$$\boldsymbol{Q}_l = \boldsymbol{I}_{N_{\rm s}} \otimes \widehat{\boldsymbol{F}}_l - (\boldsymbol{\Phi}_l \boldsymbol{\Phi}_0^{-1}) \otimes \boldsymbol{I}_{N_{\rm r}}. \tag{19}$$

By stacking $\boldsymbol{Q}_1, \boldsymbol{Q}_2, \ldots, \boldsymbol{Q}_{L-1}$ together, we define

$$\boldsymbol{Q} \triangleq \left[\boldsymbol{Q}_1^T, \boldsymbol{Q}_2^T, \cdots, \boldsymbol{Q}_{L-1}^T\right]^T. \tag{20}$$

where $\boldsymbol{Q}$ has $(L-1)N_{\rm r}N_{\rm s}$ rows and $N_{\rm r}N_{\rm s}$ columns.

Based on (17), we have

$$\boldsymbol{Q}\boldsymbol{h} = \boldsymbol{0} \tag{21}$$

Algorithm 1 Initial Channel Estimation for RIS aided massive MIMO system

Input: $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{\Phi}$, ϵ.
Obtain $\widehat{\boldsymbol{F}}_l, l = 1, 2, \ldots, L-1$ via (10).
Obtain $\boldsymbol{Q}_l, l = 1, 2, \ldots, L-1$ via (19).
Obtain $\boldsymbol{Q}$ via (20).
Obtain $\boldsymbol{h}$ via (21).
Reshape $\boldsymbol{h}$ as $\boldsymbol{H}_0$ via (16).
Set $k \leftarrow 0$.
while (14) is not satisfied **do**
  Obtain $\boldsymbol{H}_{k+1}$ by (13).
  $k \leftarrow k+1$.
end while
Set $\widehat{\boldsymbol{H}} = \boldsymbol{H}_k$.
Obtain $\widehat{\boldsymbol{G}}$ via (26).
Ouput: $\widehat{\boldsymbol{H}}$, $\widehat{\boldsymbol{G}}$.

which can be solved by the existing methods, such as eigenvalue decomposition or singular value decomposition. Obviously, zero vector is a solution of $\boldsymbol{h}$ to (21), but it is not the expected result. The other solutions can be used as $\mathrm{vec}(\boldsymbol{H})$ to initialize the iteration of (13).

For $L = 2$, the first $N_{\rm r}$ rows and $N_{\rm r}$ columns of $\boldsymbol{Q}_1$ in (18) can be rewritten as

$$(\boldsymbol{F}_1 - e^{j(\theta_1(1) - \theta_1(0))} \boldsymbol{I}_{N_{\rm r}}) \boldsymbol{H} = \boldsymbol{H} \boldsymbol{\Phi_0} \boldsymbol{\Phi_1^{-1}} - e^{j(\theta_1(l) - \theta_1(0))} \boldsymbol{H}. \tag{22}$$

We can see that the first column vector of the right side of (22) is a zero vector, which means the left side of (22) is not a full rank matrix. Since $\boldsymbol{H}$ is a full column rank matrix, $\boldsymbol{F}_1 - e^{j(\theta_1(1) - \theta_1(0))} \boldsymbol{I}_{N_{\rm r}}$ is not a full rank matrix, with the rank no larger than $N_{\rm r} - 1$. Consequently, the rank of $\boldsymbol{Q}$ is no larger than $(N_{\rm r} - 1)N_{\rm s}$, which implies that we have no more than $(N_{\rm r} - 1) \times N_{\rm s}$ independent linear equations. Therefore, we cannot determine $\boldsymbol{h}$ based on (21). It is required that

$$L \geq 3. \tag{23}$$

After we obtain the solution to (21), it can be used as the initial value of the iteration (13) and estimate $\boldsymbol{H}$. Once $\boldsymbol{H}$ is estimated, we estimate $\boldsymbol{G}$ as follows. We define

$$\boldsymbol{Y} \triangleq \left[\boldsymbol{Y}_0^T, \boldsymbol{Y}_1^T, \cdots, \boldsymbol{Y}_{L-1}^T\right]^T, \tag{24}$$

$$\boldsymbol{\Phi} \triangleq \left[\boldsymbol{\Phi}_0^T \widehat{\boldsymbol{H}}^T, \boldsymbol{\Phi}_1^T \widehat{\boldsymbol{H}}^T, \cdots, \boldsymbol{\Phi}_{L-1}^T \widehat{\boldsymbol{H}}^T\right]^T \tag{25}$$

where $\widehat{\boldsymbol{H}}$ is an estimate of $\boldsymbol{H}$. Based on (2), we can estimate $\boldsymbol{G}$ by LS as

$$\widehat{\boldsymbol{G}} = (\boldsymbol{\Phi}^H \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^H \boldsymbol{Y} \boldsymbol{X}^H (\boldsymbol{X}\boldsymbol{X}^H)^{-1}. \tag{26}$$

The detailed steps are summarized in **Algorithm** 1.

$$\begin{cases} \widehat{\boldsymbol{F}}_l(i,1)\boldsymbol{H}(1,1) + \widehat{\boldsymbol{F}}_l(i,2)\boldsymbol{H}(2,1) + \cdots + \widehat{\boldsymbol{F}}_l(i,N_\mathrm{r})\boldsymbol{H}(N_\mathrm{r},1) = \boldsymbol{H}(i,1)e^{j(\theta_1(l)-\theta_1(0))} \\ \widehat{\boldsymbol{F}}_l(i,1)\boldsymbol{H}(1,2) + \widehat{\boldsymbol{F}}_l(i,2)\boldsymbol{H}(2,2) + \cdots + \widehat{\boldsymbol{F}}_l(i,N_\mathrm{r})\boldsymbol{H}(N_\mathrm{r},2) = \boldsymbol{H}(i,2)e^{j(\theta_2(l)-\theta_2(0))} \\ \vdots \\ \widehat{\boldsymbol{F}}_l(i,1)\boldsymbol{H}(1,N_\mathrm{s}) + \widehat{\boldsymbol{F}}_l(i,2)\boldsymbol{H}(2,N_\mathrm{s}) + \cdots + \widehat{\boldsymbol{F}}_l(i,N_\mathrm{r})\boldsymbol{H}(N_\mathrm{r},N_\mathrm{s}) = \boldsymbol{H}(i,N_\mathrm{s})e^{j(\theta_{N_\mathrm{s}}(l)-\theta_{N_\mathrm{s}}(0))} \end{cases} \quad (15)$$

$$\boldsymbol{Q}_l = \begin{bmatrix} \widehat{\boldsymbol{F}}_l - e^{j(\theta_1(l)-\theta_1(0))}\boldsymbol{I}_{N_\mathrm{r}} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \widehat{\boldsymbol{F}}_l - e^{j(\theta_2(l)-\theta_2(0))}\boldsymbol{I}_{N_\mathrm{r}} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \widehat{\boldsymbol{F}}_l - e^{j(\theta_{N_\mathrm{s}}(l)-\theta_{N_\mathrm{s}}(0))}\boldsymbol{I}_{N_\mathrm{r}} \end{bmatrix} \quad (18)$$

B. Regular Channel Estimation

In practice, the RIS and the BS are geographically fixed. The RIS may be placed in an open area so that the channel condition between the BS and the RIS is relatively good, e.g., there is line-of-sight link between the BS and the RIS. Therefore, we do not need to frequently estimate $\boldsymbol{H}$. On the contrary, we may frequently estimate the channels linked to the UE, due to the movement of the UE. In this context, it is essential to estimate $\boldsymbol{G}$ and $\boldsymbol{J}$, given $\widehat{\boldsymbol{H}}$, which can be expressed as

$$\boldsymbol{Y}_l = \big(\widehat{\boldsymbol{H}}\boldsymbol{\Phi}_l\boldsymbol{G} + \boldsymbol{J}\big)\boldsymbol{X} + \boldsymbol{N}_l. \quad (27)$$

Since the estimation of $\boldsymbol{G}$ and $\boldsymbol{J}$ are decoupled in initial channel estimation, the problem of (27) is much easier than that of (2). Note that only transmitting pilots without any change of the phases of the RIS cannot estimate $\boldsymbol{G}$ and $\boldsymbol{J}$, since $\boldsymbol{G}$ and $\boldsymbol{J}$ are overlapped according to (27).

We resort to the assistance of phase changes from the RIS, which can provide additional channel measurements. Suppose we use multiple $\boldsymbol{\Phi}_l, l = 0, 1, \ldots, L-1$, where $L \geq 2$. Without loss of generality, we select $\boldsymbol{\Phi}_0$ as the reference. We have

$$\boldsymbol{Y}_0 = \widehat{\boldsymbol{H}}\boldsymbol{\Phi}_0\boldsymbol{G}\boldsymbol{X} + \boldsymbol{J}\boldsymbol{X} + \boldsymbol{N}_0, \quad (28)$$

$$\boldsymbol{Y}_n = \widehat{\boldsymbol{H}}\boldsymbol{\Phi}_n\boldsymbol{G}\boldsymbol{X} + \boldsymbol{J}\boldsymbol{X} + \boldsymbol{N}_n, n = 1, 2, \ldots, L-1. \quad (29)$$

By subtracting (29) from (28), we can remove the unknown term of $\boldsymbol{J}\boldsymbol{X}$ and have

$$\boldsymbol{Y}_0 - \boldsymbol{Y}_n = \widehat{\boldsymbol{H}}(\boldsymbol{\Phi}_0 - \boldsymbol{\Phi}_n)\boldsymbol{G}\boldsymbol{X} + \boldsymbol{N}_0 - \boldsymbol{N}_n. \quad (30)$$

for $n = 1, 2, \ldots, L-1$. By stacking them together, we have

$$\boldsymbol{Y}_\mathrm{stack} = \boldsymbol{B}\boldsymbol{G}\boldsymbol{X} + \boldsymbol{N}_\mathrm{stack} \quad (31)$$

where

$$\begin{aligned} \boldsymbol{Y}_\mathrm{stack} &\triangleq \big[(\boldsymbol{Y}_0 - \boldsymbol{Y}_1)^T, (\boldsymbol{Y}_0 - \boldsymbol{Y}_2)^T, \ldots, (\boldsymbol{Y}_0 - \boldsymbol{Y}_{L-1})^T\big]^T \\ \boldsymbol{N}_\mathrm{stack} &\triangleq \big[(\boldsymbol{N}_0 - \boldsymbol{N}_1)^T, (\boldsymbol{N}_0 - \boldsymbol{N}_2)^T, \ldots, (\boldsymbol{N}_0 - \boldsymbol{N}_{L-1})^T\big]^T \\ \boldsymbol{B} &\triangleq \big[(\boldsymbol{\Phi}_0 - \boldsymbol{\Phi}_1)^T\widehat{\boldsymbol{H}}^T, (\boldsymbol{\Phi}_0 - \boldsymbol{\Phi}_2)^T\widehat{\boldsymbol{H}}^T, \ldots, \\ &\qquad (\boldsymbol{\Phi}_0 - \boldsymbol{\Phi}_{L-1})^T\widehat{\boldsymbol{H}}^T\big]^T. \end{aligned} \quad (32)$$

It is seen from (31) that the estimation of $\boldsymbol{G}$ needs the right inverse matrix of $\boldsymbol{X}$, which means that

$$P \geq N_\mathrm{t}. \quad (33)$$

so that the columns of $\boldsymbol{X}$ are more than its rows. Note that (33) and (9) are independent to each other, since they correspond to two different parts, i.e., regular channel estimation and initial channel estimation, respectively. The noise term can be smoothed and neglected if larger L is used. Then an estimate of $\boldsymbol{G}$ based on (31) can be written as

$$\widetilde{\boldsymbol{G}} = (\boldsymbol{B}^H\boldsymbol{B})^{-1}\boldsymbol{B}^H\boldsymbol{Y}_\mathrm{stack}\boldsymbol{X}^H(\boldsymbol{X}\boldsymbol{X}^H)^{-1}. \quad (34)$$

Once $\boldsymbol{G}$ is estimated, $\boldsymbol{J}$ can be fast computed based on (28) or (29).

In fact, the uplink channel estimation is better than the downlink channel estimation for the RIS aided MIMO wireless system. During the uplink channel estimation, the BS is aware of the phase change of the RIS, since the BS is connected in wire to the RIS. The BS is capable of changing the RIS to acquire more measurements of the channel. However, in the downlink channel estimation, the UE may not know the phase change of the RIS as there is no wired connection between the RIS and the UE, which makes difficult to acquire more measurements by changing the phase of the RIS.

IV. Simulation Results

Now we evaluate the channel estimation performance for the RIS aided massive MIMO system. We set $N_\mathrm{r} = 64$, $N_\mathrm{s} = 64$, $N_\mathrm{t} = 16$ and $\sigma_1 = \sigma_2 = \sigma_3 = 1$. We use the BPSK modulation, where the pilot symbols, i.e., the entry of $\boldsymbol{X}$, is randomly selected from $\{-1, 1\}$. The normalized mean squared error (NMSE) is defined as

$$\mathrm{NMSE} = \mathbb{E}\left[\frac{\|\widehat{\boldsymbol{H}} - \boldsymbol{H}\|_F^2}{\|\boldsymbol{H}\|_F^2}\right] \quad (35)$$

where $\widehat{\boldsymbol{H}}$ is an estimate of $\boldsymbol{H}$.

As shown in Fig. 2, we simulate the convergence of the proposed initial channel estimation algorithm. We set the signal-to-noise ratio (SNR) to be 10dB, $L = 3$ and $\epsilon = 0.01$. It is seen that the initial value of the iteration is critical, where the selected $\boldsymbol{H}_0$ leads to much better performance of the estimation than the random $\boldsymbol{H}_0$. Moreover, using more pilots lead to better channel estimation performance. When doubling the pilots from $P = 64$ to $P = 128$, we can achieve around 4.5dB improvement in terms of NMSE.

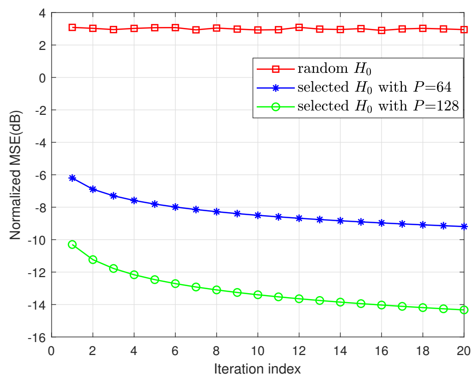

Fig. 2. Convergence of the initial channel estimation algorithm.

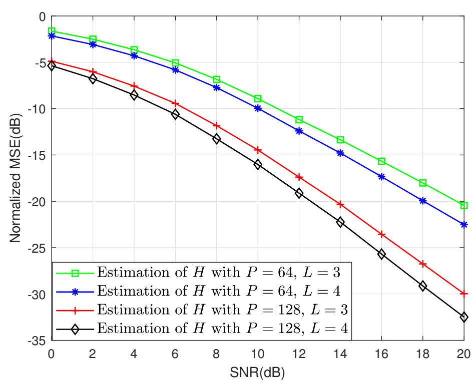

Fig. 3. Performance comparisons for initial channel estimation.

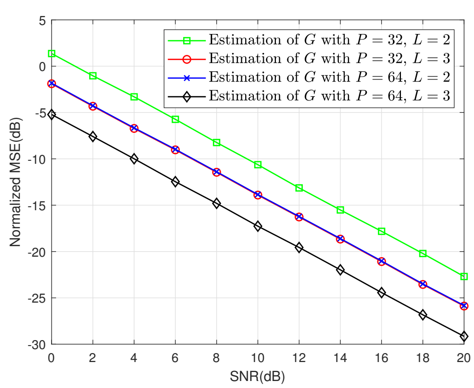

Fig. 4. Performance comparisons for regular channel estimation.

As shown in Fig. 3, we provide performance comparisons for initial channel estimation. We set $\epsilon = 0.01$. It is seen that the performance improvement of $P = 128, L = 3$ over $P = 64, L = 3$ is larger than that of $P = 63, L = 4$ over $P = 64, L = 3$, which implies that enlarging the number of pilots is more effective than increasing the RIS phase shifts. In fact, enlarging the number of pilots leads to better estimation of $\widehat{\boldsymbol{F}}_l$ in (10), while increasing the RIS phase shifts cannot yield better estimation of $\widehat{\boldsymbol{F}}_l$ but can get better estimation of $\boldsymbol{h}$ based on $\widehat{\boldsymbol{F}}_l$ according to (17).

As shown in Fig. 4, we provide performance for regular channel estimation. We assume that the estimation of $\boldsymbol{H}$ is ideal in initial channel estimation. It is seen that setting $P = 64, L = 2$ leads to the same performance as setting $L = 3, P = 32$. In fact, doubling $L - 1$, e.g., from $L = 2$ to $L = 3$, is the same as doubling P in terms of the estimation performance, according to (34). Moreover, if we only double the number of pilots, there will be around 3dB performance improvement.

V. Conclusion

In this work, we have investigated the uplink channel estimation for the RIS aided massive MIMO system. By changing the phase matrix of the RIS, we have obtained multiple channel measurements for the channel estimation. Future work will be continued with the focus on the low-complexity channel estimation algorithms.

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