Weighted Sum-Rate Maximization for Analog Beamforming and Combining in Millimeter Wave Massive MIMO Communications

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***Abstract*— The analog beamforming and analog combining in millimeter-wave (mm-Wave) massive MIMO communications are investigated. Based on a hierarchical codebook, the weighted sum-rate maximization is studied by jointly considering the duration for channel training and the receiving signal-to-noise ratio (SNR). An algorithm employing exhaustive search at the starting level of the codebook and the multi-sectional search at the other levels is proposed. At each iteration, the channel gain of the dominant path is estimated and the level of the codebook achieving the weighted sum-rate maximization is predicted by the receiver. Then the predicted level is fed back to the transmitter. If the predicted level is reached by the algorithm, the transmitter begins the data transmission with the obtained analog beamformer and the receiver uses the obtained analog combiner for data receiving. Simulation results show that the proposed algorithm outperforms multi-sectional search, where the former can achieve the weighted sum-rate almost twice of the latter when each block is consisted of 35 symbols and the channel SNR is 20 dB.**

***Index Terms*— Millimeter wave communications, analog beamforming, massive MIMO, sum-rate maximization.**

I. INTRODUCTION

MILLIMETER wave (mmWave) massive MIMO communication is one promising technology in next-generation wireless communications due to its abundant spectrum resource which is scarce and costly today. However, one disadvantage of mmWave communications is the high path loss during the transmission. To compensate the path loss, mmWave communication systems usually use directional beamforming and combining by employing large antenna arrays.

Typically, the transmitter of mmWave massive MIMO communication includes a digital baseband beamformer and an analog RF beamformer, while the receiver includes an analog RF combiner and a digital baseband combiner correspondingly [1]. The directional beamforming and receiving are mainly carried out by the analog beamformer and analog combiner, which are expected to pointed at the physically angle of departure (AOD) and physically angle of arrival (AOA) of the channel, respectively. However, the accuracy of AOD and AOA relies on channel training before data transmission.

Currently there are two kinds of methods for mmWave channel training, including beamspace channel estimation [2] and analog beam training [3], [4]. Beamspace channel estimation explores the sparse property of mmWave channel and formulates it as a sparse recovery problem. Analog beam training usually establishes two codebooks respectively at transmitter and receiver, and then sequentially takes the codeword from the codebook as beamformer for transmitter or combiner for receiver until a pair of codewords most matched with the channel AOD and AOA is found. Considering that the exhaustive search of codewords is inefficient in terms of both time and cost, a hierarchical codebook consisting of a small number of low-resolution codewords covering wide angle at top level of the codebook and a large number of high-resolution codewords offering high directional beamforming gain at bottom level of the codebook is proposed. To efficiently search from the hierarchical codebook, multi-sectional search is adopted [4].

In this letter, based on the hierarchical codebook, we study the weighted sum-rate maximization by jointly considering the duration for channel training and the receiving signal-to-noise ratio (SNR). We propose an algorithm employing exhaustive search at the starting level of the codebook and the multi-sectional search at the other levels. At each iteration, the receiver estimates the channel gain of the dominant path and predicts the level of the codebook achieving the weighted sum-rate maximization. Then the predicted level is fed back to the transmitter. If the predicted level is reached by the algorithm, the transmitter begins the data transmission with the obtained analog beamformer and the receiver uses the obtained analog combiner for data receiving.

The notations are defined as follows. Symbols for matrices (upper case) and vectors (lower case) are in boldface. $(\cdot)^H$, $\circ$, $\lceil\cdot\rceil$, $\lfloor\cdot\rfloor$, $\boldsymbol{I}_N$, $\mathrm{E}\{\cdot\}$, $\|\cdot\|_2$ and $\mathcal{CN}$ denote the conjugate transpose (Hermitian), the entry-wise product, ceiling operation, flooring operation, the identity matrix of size N, the expectation, ℓ_2 norm and the complex Gaussian distribution, respectively. $\mathbb{C}$ denotes the set of complex numbers.

II. SYSTEM MODEL

We consider an mmWave massive MIMO system which has N_t transmit antennas and N_r receive antennas. Antennas at both the transmitter and the receiver are equipped as uniform linear arrays (ULAs) with half wavelength interval. Typically, the transmitter includes a digital baseband beamformer and an analog RF beamformer, while the receiver includes an analog RF combiner and a digital baseband combiner correspondingly. We consider single data stream transmission, therefore we focus on the design of analog beamformer and analog combiner. The system can be modeled as

$$r = \sqrt{P}\mathbf{f}_r^H \mathbf{H}\mathbf{f}_t s + \mathbf{f}_r^H \boldsymbol{\eta} \tag{1}$$

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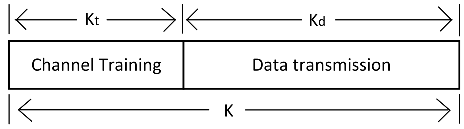

Fig. 1. Block structure for mmWave transmission.

where $\mathbf{f}_t \in \mathbb{C}^{N_t}$, $\mathbf{f}_r \in \mathbb{C}^{N_r}$ and $\boldsymbol{\eta} \in \mathbb{C}^{N_r}$ are the beamformer, the combiner, an additive white Gaussian noise vector with $\boldsymbol{\eta} \sim \mathcal{CN}(0, \sigma_\eta^2 \boldsymbol{I}_{N_r})$, respectively. P denotes the total power of transmitter. s and r denote the symbol input to the analog beamformer and the symbol output from the analog combiner, respectively. It is common to assume that the power of transmit symbol, the power of beamformer and combiner are all normalized, i.e., $\mathrm{E}\{|s|^2\} = 1$, $\|\mathbf{f}_t\|_2 = 1$ and $\|\mathbf{f}_r\|_2 = 1$. The channel matrix is defined as [5]

$$\mathbf{H} = \sqrt{N_t N_r} \sum_{l=1}^{L} g_l \mathbf{u}(N_r, \theta_l^r) \mathbf{u}(N_t, \theta_l^t)^H \tag{2}$$

where L and $g_l \sim \mathcal{CN}(0, \sigma_g^2)$ denote the number of multi-path and the channel gain of the $l(l = 1, 2, \ldots, L)$th path, respectively. $\mathbf{u}(N_t, \theta_l^t)$ and $\mathbf{u}(N_r, \theta_l^r)$ denote steering vectors at transmitter and receiver, respectively, where

$$\mathbf{u}(N, \theta) \triangleq \frac{1}{\sqrt{N}} [1, e^{j\pi\theta}, \ldots, e^{j\pi(N-1)\theta}]^T. \tag{3}$$

Define the physical angle of departure (AOD) and the physical angle of arrival (AOA) of the lth path as ϕ_l^t and ϕ_l^r, respectively. Further define $\theta_l^t = \frac{2d}{\lambda} \sin \phi_l^t$ and $\theta_l^r = \frac{2d}{\lambda} \sin \phi_l^r$, where $d \triangleq \frac{\lambda}{2}$ with λ representing the wavelength of mmWave signal. Since $\phi_l^t \in (-\frac{\pi}{2}, \frac{\pi}{2})$ and $\phi_l^r \in (-\frac{\pi}{2}, \frac{\pi}{2})$, we have $\theta_l^t \in (-1, 1)$ and $\theta_l^r \in (-1, 1)$.

III. Problem Formulation

Assume that we start the mmWave transmission in unit of blocks and the length of each block is shorter than the channel coherence time, e.g, $g_l (l = 1, 2, \ldots, L)$ can be considered to be constant during the transmission of each block. As shown in Fig. 1, each block consisted of K symbols is divided into K_t symbols for channel training and K_d symbols for data transmission, i.e., $K = K_t + K_d$.

Compared to directly maximizing the sum-rate, it is better to maximize the weighted sum-rate, regarding that the channel training does not contribute to the sum-rate [6]. The weighted sum-rate maximization problem can be formulated as

$$\max_{K_t, \sigma_x^2} \frac{K - K_t}{K} \log_2 \left(1 + \frac{\sigma_x^2}{\sigma_\eta^2}\right) \tag{4}$$

where $\sigma_\eta^2 = \mathrm{E}\{|\mathbf{f}_r^H \boldsymbol{\eta}|^2\}$ is average noise power and $\sigma_x^2 = \mathrm{E}\{|\sqrt{P} \mathbf{f}_r^H \mathbf{H} \mathbf{f}_t s|^2\}$ is average signal power. To make (4) large, K_t is expected to be small while σ_x is expected to be large. However, large σ_x implying high directional gain with high-resolution codewords requires long time of channel training or codebook search, which leads to large K_t. Therefore, the joint design of K_t and σ_x is the concentration of this work.

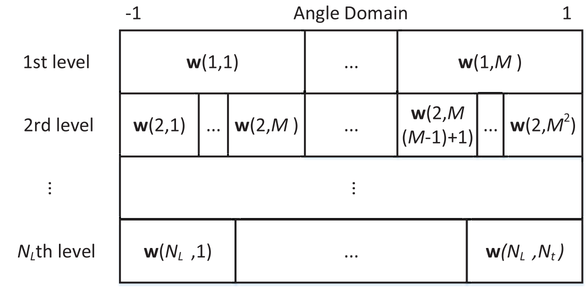

Fig. 2. Hierarchical codebook.

Now we consider a hierarchical codebook $\mathbf{W} \in \mathbb{C}^{N_t \times N_w}$ with $N_w \triangleq \lceil M(N_t - 1)/(M - 1) \rceil$ codewords at transmitter. Usually N_t is supposed to be an integer order of M, e.g., $N_t = 64$, $M = 4$. As shown in Fig. 2, the hierarchical codebook includes $N_L \triangleq \log_M N_t$ levels. The $k(k = 1, 2, \ldots, M^m)$th codeword at $m(m = 1, 2, \ldots, N_L)$th level is denoted as $\mathbf{w}(m, k)$, which is essentially the $((M^m - M)/(M - 1) + k)$th column of $\mathbf{W}$. At the first level of the codebook, the angle domain $(-1, 1)$ is divided into M codewords $\mathbf{w}(1, k)$, $k = 1, 2, \ldots, M$. At the $m(m = 1, 2, \ldots, N_L - 1)$ level, the $k(k = 1, 2, \ldots, M^m)$th codeword $\mathbf{w}(m, k)$ is further divided into M leaf codewords $\mathbf{w}(m + 1, M(k - 1) + i)$, $i = 1, 2, \ldots, M$, which are the codewords at the $(m + 1)$th level. Each codeword can be used as an analog beamformer $\mathbf{f}_t$ in (1) to produce directional transmission. Compared to the codewords at lower level, the codewords at higher level can achieve narrower main lobe of beam while the directional gain is larger. There are N_t codewords with the highest directional gain at the N_Lth level of the codebook.

Two conditions are commonly used for the design of codewords [7]. **1)** For the codewords $\mathbf{w}(m, k)$, $k = 1, 2, \ldots, M^m$ at the same $m(m = 1, 2, \ldots, N_L)$th level, no matter which codeword covers the steering vector $\mathbf{u}(N_t, \theta_l^t)$, the directional gain is the same $\sqrt{C_m^t}$. **2)** If $\mathbf{u}(N_t, \theta_l^t)$ is not covered by the codeword, the directional gain tends to zero. These two conditions can be formulated as

$$\left| \mathbf{u}(N_t, \theta_l^t)^H \mathbf{w}(m, k) \right| = \begin{cases} \sqrt{C_m^t} & \theta_l^t \in \alpha_t(m, k) \\ 0 & \theta_l^t \notin \alpha_t(m, k) \end{cases} \tag{5}$$

where $\alpha_t(m, k) = (-1 + \frac{2k-2}{M^m}, -1 + \frac{2k}{M^m})$ is the main lobe of $\mathbf{w}(m, k)$ in angle domain.

Similarly, we can establish a hierarchical codebook $\mathbf{V} \in \mathbb{C}^{N_r \times N_v}$ with $N_v \triangleq \lceil M(N_r - 1)/(M - 1) \rceil$ codewords at receiver. The codebook includes $N_J \triangleq \log_M N_r$ levels. The $i(i = 1, 2, \ldots, M^n)$th codeword at $n(n = 1, 2, \ldots, N_J)$th level is denoted as $\mathbf{v}(n, i)$, which is essentially the $((M^n - M)/(M - 1) + i)$th column of $\mathbf{V}$. Each codeword of $\mathbf{V}$ can be used as an analog combiner $\mathbf{f}_r$ in (1) to produce directional receiving. Corresponding to (5), we have

$$\left| \mathbf{u}(N_r, \theta_l^r)^H \mathbf{v}(n, i) \right| = \begin{cases} \sqrt{C_n^r} & \theta_l^r \in \alpha_r(n, i) \\ 0 & \theta_l^r \notin \alpha_r(n, i) \end{cases} \tag{6}$$

where $\alpha_r(n, i) = (-1 + \frac{2i-2}{M^n}, -1 + \frac{2i}{M^n})$ is the main lobe of $\mathbf{v}(n, i)$ in angle domain.

In mmWave channel, there is usually one dominant path and several secondary paths. We assume the $l_1(l_1 \in \{1, 2, \ldots, L\})$th path is dominant while the other $L-1$ paths are secondary. Suppose that the dominant path and the secondary paths can be distinguished by the codewords in the mth level and nth level at transmitter and receiver, respectively. If $\theta_{l_1}^t \in \alpha_t(m,k)$ and $\theta_{l_1}^r \in \alpha_r(n,i)$, the joint directional gain can be expressed as

$$\left|\mathbf{v}(n,i)^H \mathbf{u}(N_r, \theta_l^r)\mathbf{u}(N_t, \theta_l^t)^H \mathbf{w}(m,k)\right| = \sqrt{C_n^r C_m^t}. \quad (7)$$

Define the channel SNR as $\gamma = P/\sigma_\eta^2$. Then the receiving SNR in (4) can be formulated as $\sigma_x^2/\sigma_\eta^2 = \gamma N_t N_r C_n^r C_m^t |g_{l_1}|^2$. So (4) can be rewritten as

$$\max_{K_t,m,n} \frac{K-K_t}{K} \log_2\left(1+\gamma N_t N_r C_n^r C_m^t |g_{l_1}|^2\right) \quad (8)$$

where K, N_t, N_r and γ are determined before channel training. K_t depends on the structure of codebooks as well as the search method used for the codebooks. C_m^t and C_n^r depend on the searched level m for $\mathbf{W}$ and n for $\mathbf{V}$, which are the finally reached level by the search method and the corresponding codewords will be used as $\mathbf{f}_t$ and $\mathbf{f}_r$ for data transmission and receiving. Note that the codebook $\mathbf{W}$ and $\mathbf{V}$ are established and known to both the transmitter and the receiver before channel training. Therefore, to maximize the weighted sum-rate in (8), we need to find proper m, n and K_t.

IV. Analog Beamforming Design Based on Codebook

Once the codebook $\mathbf{W}$ and $\mathbf{V}$ are established, it is important to figure out a search algorithm starting from low level to high level to efficiently find out a pair of codewords best matched with the channel steering vector. As shown in Fig. 2, given $\mathbf{w}(m,k)$, only its M leaf codewords $\mathbf{w}(m+1, M(k-1)+p)$, $p = 1, 2, \ldots, M$ will be tested. Similarly at receiver, given $\mathbf{v}(n,i)$, only its M leaf codewords $\mathbf{v}(n+1, M(i-1)+q)$, $q = 1, 2, \ldots, M$ will be tested. A straightforward method is letting the transmitter use $\mathbf{f}_t = \mathbf{w}(m+1, M(k-1)+p)$, $p = 1, 2, \ldots, M$ and the receiver use $\mathbf{f}_r = \mathbf{v}(n+1, M(i-1)+q), q = 1, 2, \ldots, M$ so that the best pair of codewords can be found. However, it requires testing M^2 pairs of codewords at each level. To improve the efficiency of testing, it is better to use multi-sectional search. We first fix $\mathbf{f}_t = \mathbf{w}(m,k)$ and let $\mathbf{f}_r = \mathbf{v}(n+1, M(i-1)+q)$, $q = 1, 2, \ldots, M$ so that we can find a best $\mathbf{f}_r$, e.g., $\mathbf{f}_r = \mathbf{v}(n+1, i^*)$. Then we fix $\mathbf{f}_r = \mathbf{v}(n+1, i^*)$ and let $\mathbf{f}_t = \mathbf{w}(m+1, M(k-1)+p)$, $p = 1, 2, \ldots, M$ so that we can find a best $\mathbf{f}_t$, e.g., $\mathbf{f}_t = \mathbf{w}(m+1, k^*)$. Therefore, the multi-sectional search only needs testing $2M$ pairs of codewords at each level.

Nevertheless, all existing search algorithms search the codebook till the highest level, leading to large K_t. It is possible that some level other than the highest level can achieve the weighted sum-rate maximization. Now we propose an algorithm to maximize the weighted sum-rate for analog beamforming and combining.

We assume that $N_t \geq N_r$. Suppose that the receiver starts the search from the $m_F(m_F = 1, 2, \ldots, N_J)$th level of $\mathbf{V}$. To make sure that the transmitter and the receiver can finish the search of codebook simultaneously, we let the transmitter start the search from the $(m_F + \Delta m)$th level of $\mathbf{W}$, where $\Delta m \triangleq N_L - N_J$.

The proposed algorithm uses the exhaustive search at the m_Fth level and the multi-sectional search at the other levels. Note that we can only use the exhaustive search at the m_Fth level because we have no prior knowledge of the previous levels. From Step 3 to Step 9, we exhaustively search from $M^{2m_F+\Delta m}$ pairs of codewords at the m_Fth level to find the best one. From Step 14 to Step 23, we employ the multi-sectional search method to search from only $2M$ pairs of codewords to fast find the best one at the $\Gamma(\Gamma > m_F)$th level. In terms of each test of a pair of codewords at the mth level, we set $\mathbf{f}_t^{(m,p)}$ to be a codeword of $\mathbf{W}$ and $\mathbf{f}_r^{(m,q)}$ to be a codeword of $\mathbf{V}$ and then measure $r_{p,q}^{(m)}$ which is defined as

$$r_{p,q}^{(m)} = \sqrt{P}\left(\mathbf{f}_r^{(m,q)}\right)^H \mathbf{H}\mathbf{f}_t^{(m,p)} s + \left(\mathbf{f}_r^{(m,q)}\right)^H \boldsymbol{\eta} \quad (9)$$

according to (1). Then we find the best pair of p and q by $\arg\max_{p,q}\left|r_{p,q}^{(m_F)}\right|$.

Note that we estimate the channel gain of dominant path, i.e., $|g_{l_1}|$, after finishing both the exhaustive search and the multi-sectional search. At Step 10 and Step 24, we obtain an estimate of $|g_{l_1}|$ by

$$|\hat{g}_{l_1}| = \frac{\left|r_{p^*,q^*}^{(\Gamma)}\right|}{\sqrt{P N_t N_r C_\Gamma^r C_{\Gamma+\Delta m}^t}} \quad (10)$$

which is iteratively updated.

Since testing a pair of codewords requires the transmission of one training symbol, the multi-sectional search needs transmitting $2M$ training symbols at each level of codebook while the exhaustive search at the m_Fth level needs transmitting $M^{2m_F+\Delta m}$ training symbols, the number of totally transmitted training symbols after finishing the search of the $m(m \geq m_F)$th level is

$$K_t(m) = M^{2m_F+\Delta m} + 2M(m - m_F). \quad (11)$$

Substituting (11) into (8) and neglecting constant K, we have

$$\begin{aligned} m^* = \arg\max_{m\in\Phi}\Big\{ &\left(K - M^{2m_F+\Delta m} - 2M(m-m_F)\right) \\ &\cdot \log_2\left(1+\gamma N_t N_r C_m^r C_{m+\Delta m}^t |\hat{g}_{l_1}|^2\right)\Big\} \end{aligned} \quad (12)$$

where $\Phi \triangleq \{\Gamma, \Gamma+1, \ldots, N_J\}$. It is seen from (12) that given $|\hat{g}_{l_1}|$ we can predict the best level m^* achieving the weighted sum-rate maximization. Therefore, at Step 11 and Step 25, we obtain m^* via (12). After that, m^* together with p^* is fed back from the receiver to the transmitter. From Step 27 to Step 29, the receiver checks if m^* is reached by Algorithm 1 to determine the break of iterations. Once m^* is reached, the transmitter begins data transmission with the obtained analog beamformer $\mathbf{f}_t = \mathbf{w}(\Gamma + \Delta m, p^*)$ and the receiver uses the obtained analog combiner $\mathbf{f}_r = \mathbf{v}(\Gamma, q^*)$ for data receiving.

Algorithm 1 Weighted Sum-Rate Maximization Algorithm for Analog Beamforming and Combining

Input: N_t, N_r, K, P, m_F, $\mathbf{W}$, $\mathbf{V}$
Initialization: $\Delta m = N_L - N_J$
for $p = 1 : M^{m_F+\Delta m}$ **do**
  for $q = 1 : M^{m_F}$ **do**
    Set $\mathbf{f}_t^{(m_F,p)} = \mathbf{w}(m_F + \Delta m, p)$, $\mathbf{f}_r^{(m_F,q)} = \mathbf{v}(m_F, q)$.
    Receiver obtains $r_{p,q}^{(m_F)}$ via (9).
  end for
end for
Receiver computes $(p^*, q^*) = \arg\max_{p,q} |r_{p,q}^{(m_F)}|$.
Receiver sets $\Gamma = m_F$ and obtains $|\hat{g}_{l_1}|$ via (10).
Receiver obtains m^* via (12).
Receiver feeds back p^* and m^* to Transmitter.
for $\Gamma = m_F + 1 : N_J$ **do**
  for $q = M(q^* - 1) + 1 : Mq^*$ **do**
    Set $\mathbf{f}_t^{(\Gamma,p^*)} = \mathbf{w}(\Gamma - 1 + \Delta m, p^*)$, $\mathbf{f}_r^{(\Gamma,q)} = \mathbf{v}(\Gamma, q)$.
    Receiver obtains $r_{p^*,q}^{(\Gamma)}$ via (9).
  end for
  Receiver computes $q^* = \arg\max_q |r_{p^*,q}^{(\Gamma)}|$.
  for $p = M(p^* - 1) + 1 : Mp^*$ **do**
    Set $\mathbf{f}_t^{(\Gamma,p)} = \mathbf{w}(\Gamma + \Delta m, p)$, $\mathbf{f}_r^{(\Gamma,q^*)} = \mathbf{v}(\Gamma, q^*)$.
    Receiver obtains $r_{p,q^*}^{(\Gamma)}$ via (9).
  end for
  Receiver computes $p^* = \arg\max_p |r_{p,q^*}^{(\Gamma)}|$.
  Receiver updates $|\hat{g}_{l_1}|$ via (10).
  Receiver obtains m^* via (12).
  Receiver feed back p^* and m^* to Transmitter.
  if $\Gamma = m^*$ **then**
    Break.
  end if
end for
Output: $\mathbf{f}_t = \mathbf{w}(\Gamma + \Delta m, p^*)$, $\mathbf{f}_r = \mathbf{v}(\Gamma, q^*)$.

V. Simulation Results

We consider an mmWave massive MIMO communication system with $N_t = 32$ transmit antennas and $N_r = 32$ receive antennas. We set the number of multipath to be $L = 3$, where the channel AOD and AOA obey the uniform distribution $(-\pi/2, \pi/2)$. The channel gain of the dominant path is set to be $g_1 \sim \mathcal{CN}(0, 1)$, while the other two are set to be $g_2 \sim \mathcal{CN}(0, 10^{-2})$ and $g_3 \sim \mathcal{CN}(0, 10^{-2})$ [2]. We adopt the hierarchical codebook proposed in [4] for $\mathbf{W}$ and $\mathbf{V}$ with $M = 2$ and $m_F = 2$.

As shown in Fig. 3, we compare the proposed Algorithm 1 with the existing multi-sectional search method for different K and different γ in terms of weighted sum-rate. Note that the existing multi-sectional search method does not stop the search until finishing the search of the highest level of the codebook, which may lead to overlong training time and the decrease of weighted sum-rate. It is seen that Algorithm 1 outperforms multi-sectional search especially when K is small. If the channel coherence time is short,

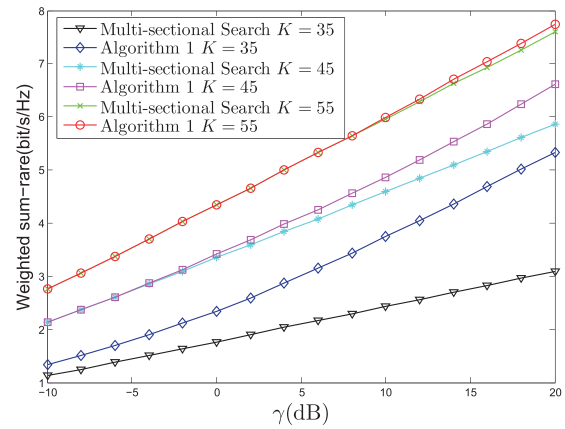

Fig. 3. Comparisons of Algorithm 1 with multi-sectional search for different K and different γ in terms of weighted sum-rate.

we have to set K small so that g_1 can be considered to be constant during the consecutive transmission of K symbols. With the increasing channel SNR indicated by γ, the gap of weighted sum-rate between Algorithm 1 and multi-sectional search becomes large, e.g., the former can be almost twice of the latter when $K = 35$ and $\gamma = 20$dB. The reason is that when the channel coherence time is not long enough, it is not optimal to search to the highest level of the codebook, since it consumes a large portion of time for channel training while the time left for data transmission becomes very small.

VI. Conclusions

In this letter, weighted sum-rate maximization for analog beamforming and analog combining has been studied by jointly considering the duration for channel training and the receiving SNR. Our future work will continue the study in multiuser scenarios by further considering digital beamforming and multiuser interference.

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