Analog Beamforming and Combining Based on Codebook in Millimeter Wave Massive MIMO Communications

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***Abstract*—Analog beamforming at transmitter and analog combining at receiver can be designed based on a hierarchical codebook, which consists of a small number of low-resolution codewords covering wide angle at top level and a large number of high-resolution codewords offering high directional beamforming gain at bottom level. Although high-resolution codewords are preferred, it takes more time for channel training. In this paper, weighted sum-rate maximization for millimeter wave (mmWave) massive MIMO communications is investigated by jointly considering the duration for channel training and the receiving signal-to-noise ratio (SNR). An algorithm is proposed to design the analog beamforming and analog combining based on the hierarchical codebook. At each iteration, the channel gain of the dominant path is estimated. The level of the hierarchical codebook achieving the weighted sum-rate maximization is predicted and then fed back to the transmitter. If the predicted level is reached, the transmitter and the receiver stop channel training and begin data transmission using the designed analog beamforming and analog combining. Simulation results show that the proposed algorithm outperforms multi-sectional search, where the former can achieve the weighted sum-rate almost twice of the latter at certain length of transmission block when the channel SNR is 20dB.**

***Index Terms*—Millimeter wave, analog beamforming, analog combining, codebook, massive MIMO.**

I. Introduction

Millimeter wave (mmWave) communication is attractive for future wireless communications due to its rich spectrum resource as well as small antenna size that can be more compactly integrated into antenna arrays [1]. In mmWave massive MIMO systems with large antenna arrays, highly directional beamforming exploiting spatial degree of freedom can be produced for efficient transmission from base station (BS) to multiple users, which is capable of substantially improving both energy efficiency and spectral efficiency.

The mmWave massive MIMO system usually employs digital baseband beamforming and analog RF beamforming at transmitter side, and analog RF combining and digital baseband combining at receiver side [2]. The directional transmitting and receiving are carried out by the analog beamforming and analog combining, which are expected to pointed at the physical angle of departure (AOD) and physical angle of arrival (AOA) of mmWave channel, respectively. The multiuser interference caused by simultaneously transmitting multiple independent data streams is tackled by digital beamforming and digital combining. Both analog beamforming and digital beamforming relies on accurate channel state information (CSI) of mmWave channel. To acquire CSI, channel training or channel estimation is usually employed before the transmission of data. Currently there are two classes of methods for mmWave channel training, including beamspace channel estimation [3] and analog beam training [4], [5]. Beamspace channel estimation explores the sparse property of mmWave channel and treats it as a sparse recovery problem. Analog beam training usually establishes two codebooks respectively at transmitter and receiver, and then sequentially takes a codeword from the codebook as analog beamforming or analog combining until a pair of codewords most matched with the channel AOD and AOA is found. Considering that the exhaustive search of codewords is inefficient in terms of both time and cost, a hierarchical codebook consisting of a small number of low-resolution codewords covering wide angle at top level of the codebook and a large number of high-resolution codewords offering high directional beamforming gain at bottom level of the codebook is proposed in [4]. Multi-sectional search is then proposed to further improve the efficiency of codebook search.

In this paper we study the weighted sum-rate maximization for mmWave massive MIMO communications by jointly considering the duration for channel training and the receiving signal-to-noise ratio (SNR). We propose an algorithm to design the analog beamforming and analog combining based on the hierarchical codebook. At each iteration, the receiver estimates the channel gain of the dominant path and predicts the level of the codebook achieving the weighted sum-rate maximization. Then the predicted level is fed back to the transmitter. If the predicted level is reached, the transmitter and the receiver stop channel training and begin data transmission using the designed analog beamforming and analog combining.

The notations are defined as follows. Symbols for matrices (upper case) and vectors (lower case) are in boldface. $\circ$, $(\cdot)^T$, $(\cdot)^H$, $\lceil\cdot\rceil$, $\lfloor\cdot\rfloor$, $\boldsymbol{I}_N$, $\mathrm{E}\{\cdot\}$, $\mathbf{0}^M$, $\|x\|_2$ and $\mathcal{CN}$ denote the entry-wise product, transpose, conjugate transpose (Hermitian), ceiling integer operation, flooring integer operation, identity matrix of size N, expectation, zero vector of size M,

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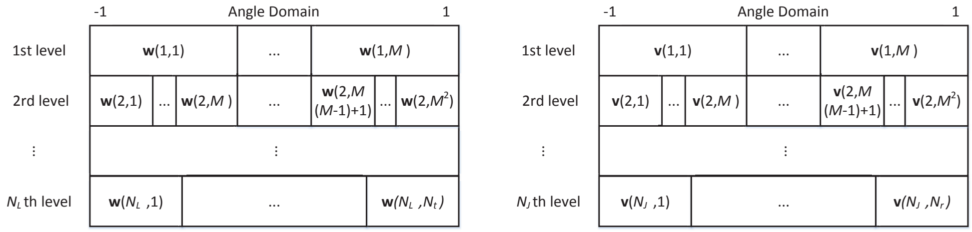

Fig. 1. Hierarchical codebook $\mathbf{W}$ and $\mathbf{V}$ for analog beamforming and analog combining respectively.

ℓ_2 norm of vector x and the complex Gaussian distribution, respectively. $\mathbb{C}$ denotes the set of complex numbers.

II. Problem Formulation

Consider a mmWave massive MIMO system including a transmitter equipped with N_t antennas and a receiver equipped with N_r antennas. Antennas at both the transmitter and the receiver are placed as uniform linear arrays (ULAs) with half wavelength interval. Typically, the transmitter employs a digital baseband beamformer and an analog RF beamformer, while the receiver employs an analog RF combiner and a digital baseband combiner correspondingly. We consider single data stream transmission, therefore we focus on the design of analog beamformer and analog combiner. The system can be modeled as

$$r = \sqrt{P}\mathbf{f}_r^H \mathbf{H}\mathbf{f}_t s + \mathbf{f}_r^H \boldsymbol{\eta} \tag{1}$$

where $\mathbf{f}_t \in \mathbb{C}^{N_t}$, $\mathbf{f}_r \in \mathbb{C}^{N_r}$ and $\boldsymbol{\eta} \in \mathbb{C}^{N_r}$ are the beamformer, the combiner, and an additive white Gaussian noise vector with $\boldsymbol{\eta} \sim \mathcal{CN}(0, \sigma_\eta^2 \boldsymbol{I}_{N_r})$, respectively. P denotes the total power of transmitter. s and r denote the symbol input to the analog beamformer and the symbol output from the analog combiner, respectively. It is common to assume that the power of transmit symbol, the power of beamformer and combiner are all normalized, i.e., $\mathrm{E}\{|s|^2\} = 1$, $\|\mathbf{f}_t\|_2 = 1$ and $\|\mathbf{f}_r\|_2 = 1$. The channel matrix can be written as [6]

$$\mathbf{H} = \sqrt{N_t N_r} \sum_{l=1}^{L} g_l \mathbf{u}(N_r, \theta_l^r)\mathbf{u}(N_t, \theta_l^t)^H \tag{2}$$

where L and $g_l \sim \mathcal{CN}(0, \sigma_g^2)$ denote the number of multi-path and the channel gain of the $l(l = 1, 2, \ldots, L)$th path, respectively. $\mathbf{u}(N_t, \theta_l^t)$ and $\mathbf{u}(N_r, \theta_l^r)$ denote steering vectors at transmitter and receiver, respectively, where

$$\mathbf{u}(N, \theta) \triangleq \frac{1}{\sqrt{N}}[1, e^{j\pi\theta}, \ldots, e^{j\pi(N-1)\theta}]^T. \tag{3}$$

Define the physical AOD and the physical AOA of the lth path as ϕ_l^t and ϕ_l^r, respectively. We have $\theta_l^t = \frac{2d}{\lambda}\sin\phi_l^t$ and $\theta_l^r = \frac{2d}{\lambda}\sin\phi_l^r$, where we usually set $d \triangleq \frac{\lambda}{2}$ with λ representing the wavelength of mmWave signal.

Assume that we start the transmission in unit of blocks and the length of each block is shorter than mmWave channel coherence time, e.g, $g_l(l = 1, 2, \ldots, L)$ can be considered to be constant during the transmission of each block. We suppose that each block consisting of K symbols is divided into K_t symbols for channel training and K_d symbols for data transmission, i.e., $K = K_t + K_d$.

Compared to directly maximizing the sum-rate, it is better to maximize the weighted sum-rate, regarding that the channel training does not contribute to the sum-rate. The weighted sum-rate maximization problem can be formulated as

$$\max_{K_t, \sigma_x^2} \frac{K - K_t}{K} \log_2\left(1 + \frac{\sigma_x^2}{\sigma_\eta^2}\right) \tag{4}$$

where $\sigma_\eta^2 = \mathrm{E}\{|\mathbf{f}_r^H \boldsymbol{\eta}|^2\}$ is average noise power and $\sigma_x^2 = \mathrm{E}\{|\sqrt{P}\mathbf{f}_r^H \mathbf{H}\mathbf{f}_t s|^2\}$ is average signal power. To maximize (4), K_t is expected to be small while σ_x is expected to be large. However, large σ_x implying high directional gain with high-resolution codewords would require long time of channel training or codebook search, which leads to large K_t and the decrease of (4). Therefore, the joint consideration of K_t and σ_x is essentially important.

III. Codebook Design

We employ analog beamforming and analog combining based on codebook. Before data transmission, two codebooks $\mathbf{W}$ and $\mathbf{V}$ as shown in Fig. 1 are established at transmitter and receiver, respectively. Then the transmitter and receiver sequentially take the codewords from the codebooks as analog beamformer and analog combiner until a pair of codewords most matched with the channel AOD and AOA is found. Considering that the exhaustive search of codewords is inefficient in terms of both time and cost, hierarchical codebooks $\mathbf{W}$ and $\mathbf{V}$ consisting of a small number of low-resolution codewords covering wide angle at top level of the codebook and a large number of high- resolution codewords offering high directional beamforming gain at bottom level of the codebook are considered.

As shown in Fig. 1, we use a hierarchical codebook $\mathbf{W} \in \mathbb{C}^{N_t \times N_w}$ with $N_w \triangleq \lceil M(N_t - 1)/(M - 1)\rceil$ codewords at transmitter. Usually N_t is supposed to be an integer order of M, e.g., $N_t = 64$, $M = 4$. Suppose $\mathbf{W}$ includes $N_L \triangleq \log_M N_t$ levels. The $k(k = 1, 2, \ldots, M^m)$th codeword at $m(m = 1, 2, \ldots, N_L)$th level is denoted as $\mathbf{w}(m, k)$, which is essentially the $((M^m - M)/(M - 1) + k)$th column of $\mathbf{W}$. At the first level (top level) of the codebook, the angle domain $(-1, 1)$ is divided into M codewords $\mathbf{w}(1, k)$, $k = 1, 2, \ldots, M$. At the $m(m = 1, 2, \ldots, N_L - 1)$ level, the $k(k = 1, 2, \ldots, M^m)$th codeword $\mathbf{w}(m, k)$ is further divided into M leaf codewords $\mathbf{w}(m + 1, M(k - 1) + i)$, $i = 1, 2, \ldots, M$, which are the codewords at the $(m+1)$th level. Each codeword can be used as an analog beamformer $\mathbf{f}_t$ in (1) to produce directional transmission. Compared to the codewords at lower level, the codewords at higher level can achieve narrower main lobe of beam while the directional gain is larger. There are N_t codewords with the highest directional gain at the N_Lth level (bottom level) of the codebook. As shown in Fig. 1, we also establish a hierarchical codebook $\mathbf{V} \in \mathbb{C}^{N_r \times N_v}$ with $N_v \triangleq \lceil M(N_r - 1)/(M - 1) \rceil$ codewords at receiver. The codebook includes $N_J \triangleq \log_M N_r$ levels. The $i(i = 1, 2, \ldots, M^n)$th codeword at $n(n = 1, 2, \ldots, N_J)$th level is denoted as $\mathbf{v}(n, i)$, which is essentially the $((M^n - M)/(M - 1) + i)$th column of $\mathbf{V}$. Each codeword of $\mathbf{V}$ can be used as an analog combiner $\mathbf{f}_r$ in (1) to produce directional receiving.

Once the codebook $\mathbf{W}$ and $\mathbf{V}$ are established, it is important to figure out a search algorithm to efficiently find out a pair of codewords best matched with the channel steering vectors. Exhaustive search is a method where the transmitter uses $\mathbf{f}_t = \mathbf{w}(N_L, k), k = 1, 2, \ldots, N_t$ and the receiver uses $\mathbf{f}_r = \mathbf{v}(N_J, i), i = 1, 2, \ldots, N_r$ so that the best pair of codewords can be found. However, it requires testing $N_t \times N_r$ pairs of codewords, leading to substantially large training time K_t and low weighted sum-rate in (4).

To improve the efficiency of codebook search, it is better to use multi-sectional search starting from low level to high level. Note that we may not start the search from the first level, since the directional gain at the first level may be not enough for mmWave signal transmission. Given $\mathbf{w}(m, k)$, only M leaf codewords $\mathbf{w}(m + 1, M(k - 1) + q)$, $q = 1, 2, \ldots, M$ will be searched. Similarly at receiver, given $\mathbf{v}(n, i)$, only M leaf codewords $\mathbf{v}(n + 1, M(i - 1) + q)$, $q = 1, 2, \ldots, M$ will be searched. In particular, we first fix $\mathbf{w}(m, k)$ to find a codeword from $\mathbf{v}(n + 1, M(i - 1) + q)$, $q = 1, 2, \ldots, M$ which can make up the best pair with $\mathbf{w}(m, k)$, e.g., $\mathbf{v}(n + 1, i^*)$. Then we fix $\mathbf{v}(n+1, i^*)$ to find a codeword from $\mathbf{w}(m+1, M(k-1) + q)$, $q = 1, 2, \ldots, M$ that can make up the best pair with $\mathbf{v}(n + 1, i^*)$. Therefore, the multi-sectional search only needs testing $2M$ pairs of codewords at each level, leading to short training time K_t and improved weighted sum-rate in (4).

Compared to the codebook design, the design of codewords is more challengeable. Two objectives are commonly used for the design of codewords [7]. **1)** For the codewords $\mathbf{w}(m, k)$, $k = 1, 2, \ldots, M^m$ at the same $m(m = 1, 2, \ldots, N_L)$th level, no matter which codeword covers the steering vector $\mathbf{u}(N_t, \theta_l^t)$, the directional gain is the same $\sqrt{C_m^t}$. **2)** If $\mathbf{u}(N_t, \theta_l^t)$ is not covered by the codeword, the directional gain tends to zero. These two conditions can be formulated as

$$|\mathbf{u}(N_t, \theta_l^t)^H \mathbf{w}(m, k)| = \begin{cases} \sqrt{C_m^t} & \theta_l^t \in \alpha_t(m, k) \\ 0 & \theta_l^t \notin \alpha_t(m, k) \end{cases} \quad (5)$$

where $\alpha_t(m, k) = (-1 + \frac{2k-2}{M^m}, -1 + \frac{2k}{M^m})$ is the main lobe of $\mathbf{w}(m, k)$ in angle domain. Similar to (5), we have

$$|\mathbf{u}(N_r, \theta_l^r)^H \mathbf{v}(n, i)| = \begin{cases} \sqrt{C_n^r} & \theta_l^r \in \alpha_r(n, i) \\ 0 & \theta_l^r \notin \alpha_r(n, i) \end{cases} \quad (6)$$

where $\alpha_r(n, i) = (-1 + \frac{2i-2}{M^n}, -1 + \frac{2i}{M^n})$ is the main lobe of $\mathbf{v}(n, i)$ in angle domain.

However, in practice it is difficult to satisfy the two objectives described in (5) and (6). The directional gain may fluctuate within the main lobe of codewords while it can not be strictly zero within the side lobe. We design the codewords of $\mathbf{W}$ and $\mathbf{V}$ based on [4]. The procedures for the design of $\mathbf{W}$ are briefly listed as follows. The design of $\mathbf{V}$ is similar.

We first design the codewords at the highest level as $\mathbf{w}(N_L, k) = \mathbf{u}(N_t, -1 + (2k - 1)/N_t)$, $k = 1, 2, \ldots, N_t$. Then the codewords at the other levels, $\mathbf{w}(m, k), m = 1, 2, \ldots, N_L - 1$, can be obtained with the following steps:

- Divide $\mathbf{w}(m, 1)$ into $a_s = M^{\lfloor (N_L - m + 1)/2 \rfloor}$ sub-arrays. Each sub-array has $b_s = N_t/a_s$ antennas. If $(N_L - m)$ is odd, $b_A = a_s/M$; otherwise, $b_A = a_s$, where b_A is defined as the number of active sub-arrays. The sub-codeword $\mathbf{y}_a$ formed by the $a(a = 1, 2, \ldots, a_s)$th sub-array is defined as

$$\mathbf{y}_a = \begin{cases} e^{ja\pi}\mathbf{u}(b_s, -1 + (2a - 1)/b_s), a = 1, 2, \ldots, b_A, \\ \mathbf{0}^{b_s}, \quad a = b_A + 1, b_A + 2, \ldots, a_s. \end{cases} \quad (7)$$

- Stack the sub-codewords $\mathbf{y}_a, a = 1, 2, \ldots, a_s$ together and obtain $\mathbf{w}(m, 1) = [\mathbf{y}_1^T, \mathbf{y}_2^T, \ldots, \mathbf{y}_{a_s}^T]^T$.
- Obtain $\mathbf{w}(m, k)$ by $\mathbf{w}(m, k) = \mathbf{w}(m, 1) \circ \sqrt{N_t}\mathbf{u}(N_t, 2(k - 1)/M^m)$, $k = 2, 3, \ldots, M^m$.
- Normalize $\mathbf{w}(m, k)$ by $\mathbf{w}(m, k)/\|\mathbf{w}(m, k)\|_2$, $k = 1, 2, \ldots, M^m$.

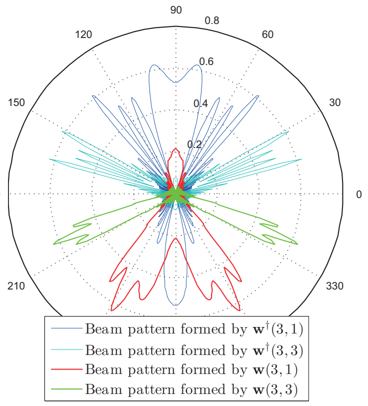

Fig. 2. Comparisons of beam patterns formed by $\mathbf{w}(3, 1)$ and $\mathbf{w}(3, 3)$ and $\mathbf{w}^\dagger(3, 1)$ and $\mathbf{w}^\dagger(3, 3)$.

As shown in Fig. 2, we compare the beam patterns formed by the codewords $\mathbf{w}(3, 1)$ and $\mathbf{w}(3, 3)$ using the above procedures with those formed by the codewords $\mathbf{w}^\dagger(3, 1)$ and $\mathbf{w}^\dagger(3, 3)$ using the methods in [8]. We set $N_t = 32$ and $M = 2$, and compare the codewords at the third level $(m = 3)$. It is seen that the beams formed by $\mathbf{w}(3, 1)$ or $\mathbf{w}(3, 3)$ are more flat than those formed by $\mathbf{w}^\dagger(3, 1)$ or $\mathbf{w}^\dagger(3, 3)$, which means $\mathbf{w}(3,1)$ and $\mathbf{w}(3,3)$ are better than $\mathbf{w}^{\dagger}(3,1)$ and $\mathbf{w}^{\dagger}(3,3)$ with respect to (5) and (6). Additionally, it is observed that there are some grooves within the main lobe of beams formed by $\mathbf{w}^{\dagger}(3,1)$ and $\mathbf{w}^{\dagger}(3,3)$. If the AOD of dominant path happens to point at any groove, the channel training will fail to detect the AOD, resulting in reduced sum-rate performance. In contrast, there is no groove within the main lobe of the beams formed by $\mathbf{w}(3,1)$ and $\mathbf{w}(3,3)$.

In mmWave channel, there is usually one line-of-sight(LOS) dominant path and several non-LOS secondary paths. We assume the $l_1 (l_1 \in \{1, 2, \ldots, L\})$th path is dominant while the other $L-1$ paths are secondary. Suppose that the dominant path and the secondary paths can be distinguished by the codewords in the mth level and nth level at transmitter and receiver, respectively. If $\theta_{l_1}^t \in \alpha_t(m,k)$ and $\theta_{l_1}^r \in \alpha_r(n,i)$, the joint directional gain can be expressed as

$$|\mathbf{v}(n,i)^H \mathbf{u}(N_r, \theta_{l_1}^r) \mathbf{u}(N_t, \theta_{l_1}^t)^H \mathbf{w}(m,k)| = \sqrt{C_n^r C_m^t}. \quad (8)$$

Note that in practice the joint directional gain in (8) may vary within the main lobe, we use the average of $|\mathbf{v}(n,i)^H \mathbf{u}(N_r, \theta_{l_1}^r) \mathbf{u}(N_t, \theta_{l_1}^t)^H \mathbf{w}(m,k)|, \theta_{l_1}^t \in (-1 + (2k-2)/M^m, -1 + 2k/M^m), \theta_{l_1}^r \in (-1 + (2i-2)/M^n, -1 + 2i/M^n)$ as $\sqrt{C_n^r C_m^t}$. Therefore, the accuracy of $\sqrt{C_n^r C_m^t}$ depends on the flatness of the joint directional gain. It is significant to design codewords with flat main lobe.

IV. Analog Beamforming Design based on Codebook

Suppose $\mathrm{E}\{|s|^2\} = 1$ for simplicity. Then the receiving SNR in (4) can be expressed as

$$\frac{\sigma_x^2}{\sigma_\eta^2} = \gamma N_t N_r C_n^r C_m^t |g_{l_1}|^2 \quad (9)$$

where $\gamma = P/\sigma_\eta^2$ is defined as the channel SNR. So (4) can be rewritten as

$$\max_{K_t, m, n} \frac{K - K_t}{K} \log_2 \left(1 + \gamma N_t N_r C_n^r C_m^t |g_{l_1}|^2\right) \quad (10)$$

where K, N_t, N_r and γ are all determined before channel training. K_t depends on the codebooks $\mathbf{W}$ and $\mathbf{V}$ as well as the search method for the codebooks. C_m^t and C_n^r depend on the searched level m for $\mathbf{W}$ and n for $\mathbf{V}$, which are the finally reached levels by the search method and the corresponding codewords will be used as $\mathbf{f}_t$ and $\mathbf{f}_r$ for data transmission and receiving. Note that the codebook $\mathbf{W}$ and $\mathbf{V}$ are established and known to both the transmitter and the receiver before channel training. Therefore, to maximize the weighted sum-rate in (10), we need to find proper m, n and K_t. However, all existing search algorithms search the codebook till the highest level, leading to large K_t. It is possible that some level other than the highest level can maximize the weighted sum-rate.

Now we propose **Algorithm 1** to design analog beamforming and analog combining to maximize the weighted sum-rate. We assume that $N_t \geq N_r$. Since the search of codebook should start from the level which has enough directional gain, we suppose that the transmitter and receiver start the search from the $m_F (1 \leq m_F \leq N_J \leq N_L)$th level of $\mathbf{W}$ and $\mathbf{V}$ simultaneously. The proposed algorithm uses exhaustive search at the m_Fth level and multi-sectional search at the other levels. From Step 3 to Step 9 of **Algorithm 1**, we exhaustively search from M^{2m_F} pairs of codewords at the m_Fth level to find the best one. To make sure that the transmitter and the receiver can finish the search of codebook simultaneously, we use the multi-sectional search solely at transmitter from the (m_F+1)th level to $(m_F + \Delta m)$th level, while the receiver stops the search and fixes on the m_Fth level, indicated by Step 11 to Step 18 of **Algorithm 1**. Then the transmitter and receiver start the multi-sectional search of codebooks simultaneously. From Step 20 to Step 29, we employ the multi-

Algorithm 1 Analog Beamforming and Combining Based on Codebook for Weighted Sum-rate Maximization

1: *Input:* N_t, N_r, K, P, m_F, $\mathbf{W}$, $\mathbf{V}$
2: *Initialization:* $\Delta m = N_L - N_J$
3: **for** $p = 1 : M^{m_F}$ **do**
4:   **for** $q = 1 : M^{m_F}$ **do**
5:     Set $\mathbf{f}_t^{(m_F,p)} = \mathbf{w}(m_F, p)$, $\mathbf{f}_r^{(m_F,q)} = \mathbf{v}(m_F, q)$.
6:     Receiver obtains $r_{p,q}^{(m_F)}$ via (11).
7:   **end for**
8: **end for**
9: Receiver computes $(p^*, q^*) = \arg\max_{p,q} |r_{p,q}^{(m_F)}|$.
10: Receiver feeds back p^* to Transmitter.
11: **for** $\Gamma = m_F + 1 : m_F + \Delta m$ **do**
12:   **for** $p = M(p^*-1)+1 : Mp^*$ **do**
13:     Set $\mathbf{f}_t^{(\Gamma,p)} = \mathbf{w}(\Gamma, p), \mathbf{f}_r^{(m_F,q^*)} = \mathbf{v}(m_F, q^*)$.
14:     Receiver obtains $r_{p,q^*}^{(\Gamma)}$ via (11).
15:   **end for**
16:   Receiver computes $p^* = \arg\max_p |r_{p,q^*}^{(\Gamma)}|$.
17:   Receiver feeds back p^* to Transmitter.
18: **end for**
19: **for** $\Gamma = m_F + \Delta m + 1 : N_L$ **do**
20:   **for** $q = M(q^*-1)+1 : Mq^*$ **do**
21:     Set $\mathbf{f}_t^{(\Gamma-1,p^*)} = \mathbf{w}(\Gamma-1, p^*)$.
    $\mathbf{f}_r^{(\Gamma-\Delta m,q)} = \mathbf{v}(\Gamma - \Delta m, q)$.
22:     Receiver obtains $r_{p^*,q}^{(\Gamma)}$ via (11).
23:   **end for**
24:   Receiver computes $q^* = \arg\max_q |r_{p^*,q}^{(\Gamma)}|$.
25:   **for** $p = M(p^*-1)+1 : Mp^*$ **do**
26:     Set $\mathbf{f}_t^{(\Gamma,p)} = \mathbf{w}(\Gamma, p), \mathbf{f}_r^{(\Gamma-\Delta m,q^*)} = \mathbf{v}(\Gamma - \Delta m, q^*)$.
27:     Receiver obtains $r_{p,q^*}^{(\Gamma)}$ via (11).
28:   **end for**
29:   Receiver computes $p^* = \arg\max_p |r_{p,q^*}^{(\Gamma)}|$.
30:   Receiver updates $|\hat{g}_{l_1}|$ via (12).
31:   Receiver obtains m^* via (14).
32:   Receiver feeds back p^* and m^* to Transmitter.
33:   **if** $\Gamma = m^*$ **then**
34:     Break.
35:   **end if**
36: **end for**
37: *Output:* $\mathbf{f}_t = \mathbf{w}(\Gamma, p^*), \mathbf{f}_r = \mathbf{v}(\Gamma - \Delta m, q^*)$.

sectional search method at both transmitter and receiver by testing $2M$ pairs of codewords to fast find the best one. In terms of each testing of a pair of codewords at the mth level, we set $\mathbf{f}_t^{(m,p)}$ to be a codeword of $\mathbf{W}$ and $\mathbf{f}_r^{(m,q)}$ to be a codeword of $\mathbf{V}$ and then measure $r_{p,q}^{(m)}$ which is defined as

$$r_{p,q}^{(m)} = \sqrt{P}\left(\mathbf{f}_r^{(m,q)}\right)^H \mathbf{H}\mathbf{f}_t^{(m,p)} s + \left(\mathbf{f}_r^{(m,q)}\right)^H \boldsymbol{\eta} \quad (11)$$

according to (1). Then we can find the best pair of p and q by $(p^*, q^*) = \arg\max_{p,q} \left| r_{p,q}^{(m_F)} \right|$.

Note that we estimate the channel gain of the dominant path, i.e., $|g_{l_1}|$, after finishing both the exhaustive search and the multi-sectional search. At Step 30, we obtain an estimate of $|g_{l_1}|$ by

$$|\hat{g}_{l_1}| = \frac{\left| r_{p^*,q^*}^{(\Gamma)} \right|}{\sqrt{P N_t N_r C_{\Gamma-\Delta m}^r C_\Gamma^t}}. \quad (12)$$

which is iteratively updated. Since testing a pair of codeword requires the transmission of one training symbol, the multi-sectional search needs transmitting M and $2M$ training symbols at each level of codebook from Step 12 to Step 16 and from Step 20 to Step 29 respectively. Considering that the exhaustive search at the m_Fth level needs transmitting M^{2m_F} training symbols, the number of totally transmitted training symbols after finishing the search of the $m(m \geq m_F)$th level is

$$K_t(m) = M^{2m_F} + M\Delta m + 2M(m - m_F - \Delta m). \quad (13)$$

Substituting (13) into (10) and neglecting constant K, we have

$$m^* = \arg\max_{m\in\Phi} \Big\{ \left(K - M^{2m_F} + M\Delta m - 2M(m - m_F)\right) \cdot \log_2\left(1 + \gamma N_t N_r C_{m-\Delta m}^r C_m^t |\hat{g}_{l_1}|^2\right) \Big\} \quad (14)$$

where $\Phi \triangleq \{\Gamma, \Gamma+1, \ldots, N_L\}$. It is seen from (14) that given $|\hat{g}_{l_1}|$ we can predict the best level m^* maximizing the weighted sum-rate. Therefore, at Step 31, we obtain m^* via (14). After that, m^* together with p^* is fed back from the receiver to the transmitter. From Step 33 to Step 35, both the transmitter and the receiver check if m^* is reached to determine the break of the iterations. Once m^* is reached, the transmitter begins data transmission with the obtained analog beamformer $\mathbf{f}_t = \mathbf{w}(\Gamma, p^*)$ and the receiver uses the obtained analog combiner $\mathbf{f}_r = \mathbf{v}(\Gamma - \Delta m, q^*)$ for data receiving. Compare to multi-sectional search, **Algorithm 1** requires additional feedback indicating if the current level is the best level maximizing the weighted sum-rate. Since it is binary feedback that only occupied one bit, the amount of additional feedback is negligible.

Additionally, we analyze the accuracy of prediction of m^*. Different with iteratively predicting m^* at different level, we compute the genuine m^o only once finishing all the steps of **Algorithm 1**. Substituting (12) into (9), we have

$$\frac{\sigma_x^2}{\sigma_\eta^2} = \frac{P}{\sigma_\eta^2} N_t N_r C_{\Gamma-\Delta m}^r C_\Gamma^t \frac{\left| r_{p^*,q^*}^{(\Gamma)} \right|^2}{P N_t N_r C_{\Gamma-\Delta m}^r C_\Gamma^t} = \frac{\left| r_{p^*,q^*}^{(\Gamma)} \right|^2}{\sigma_\eta^2} \quad (15)$$

which shows that the receiving SNR at the Γth level is uncorrelated with $|g_{l_1}|$ and the accuracy of $|\hat{g}_{l_1}|$ will not affect the receiving SNR. Further substituting (15) into (4), we have

$$m^o = \arg\max_{m\in\Psi} \left\{ \left(K - K_t(m)\right) \log_2\left(1 + \frac{\left| r_{p^*,q^*}^m \right|^2}{\sigma_\eta^2}\right) \right\}. \quad (16)$$

where $K_t(m)$ has been already provided by (13) and $\Psi \triangleq \{m_F + \Delta m, m_F + \Delta m + 1, \ldots, N_L\}$. We will compare m^* with m^o in our simulations to show the accuracy of prediction of m^*.

V. Simulation Results

We consider an mmWave massive MIMO system with $N_t = 256$ transmit antennas and $N_r = 256$ receive antennas. We set the number of multipath to be $L = 3$, where the channel AOD and AOA obey the uniform distribution $(-\pi/2, \pi/2)$. The channel gain of the dominant path is set to be $g_1 \sim \mathcal{CN}(0,1)$, while the other two are set to be $g_2 \sim \mathcal{CN}(0, 10^{-2})$ and $g_3 \sim \mathcal{CN}(0, 10^{-2})$ [3]. We adopt the hierarchical codebook presented in Section III for $\mathbf{W}$ and $\mathbf{V}$ with $M = 4$ and $m_F = 1$.

As shown in Fig. 3, we compare the proposed **Algorithm 1** with multi-sectional search for different K and different γ in terms of weighted sum-rate. Note that all existing multi-sectional search methods do not stop the search until finishing the search of the highest level of the codebook, which may lead to overlong training time and the decrease of weighted sum-rate. According to (13), multi-sectional search needs $K_t = 40$ symbols for training. It is seen that **Algorithm 1** outperforms multi-sectional search especially when K is small. If the channel coherence time is short, we have to set K small so that g_1 can be considered to be constant during the consecutive transmission of K symbols. With the increasing channel SNR indicated by γ, the gap of weighted sum-rate between **Algorithm 1** and multi-sectional search becomes large, e.g., the former can be almost twice of the latter when $K = 50$ and $\gamma = 20$dB. The reason is that when the channel coherence time is not long enough, it is not optimal to search to the highest level of the codebook, since it consumes a large portion of time for channel training while the time left for data transmission becomes very limited.

In Fig. 4, we compare $|\hat{g}_1|$ estimated by different level with $|g_1|$. We set $|g_1| = 1.13$. We fix the AOD and AOA of channel dominant path to be $\phi_1^t = -53.13°$ and $\phi_1^r = -53.13°$ so that the first term in (11) is fixed for the given m; otherwise the first term in (11) may vary with different AOD and AOA due to the fluctuation of codeword within the main lobe, resulting in the divergence of (12). In this context, the joint directional gain in (8) is $\sqrt{C_\Gamma^r C_\Gamma^t} = 0.0155, 0.0575, 0.2419, 0.9675$ for $\Gamma = 1, 2, 3, 4$, respectively. Fig. 4 shows that the estimation of $|g_1|$ at higher levels is more accurate than that at lower levels, since the codewords at higher level have higher directional gain and thus larger SNR. With increasing SNR, the estimation of $|g_1|$ fast converges to the genuine $|g_1|$. At $\gamma = 0$dB, $|\hat{g}_1|$ estimated at different levels all equals the genuine $|g_1|$.

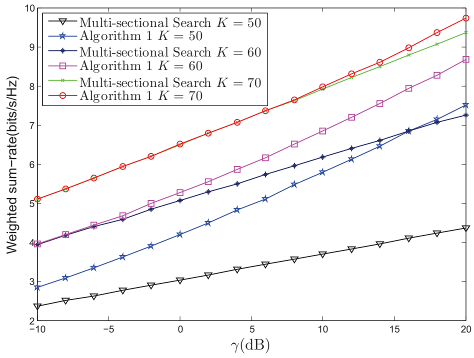

Fig. 3. Comparisons of proposed algorithm and multi-sectional search for different K and different γ in terms of weighted sum-rate.

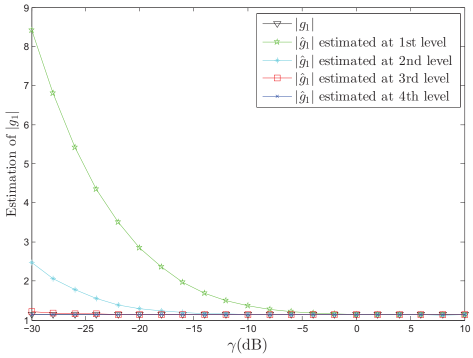

Fig. 4. Comparisons of $|\hat{g}_1|$ estimated by different levels and the genuine $|g_1|$.

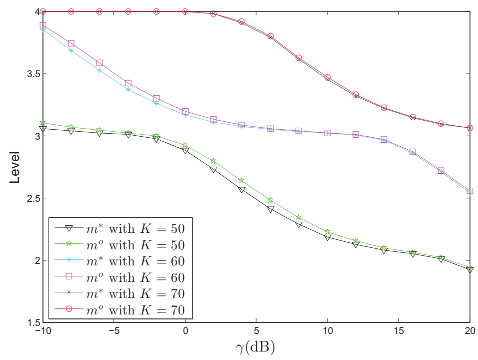

Fig. 5. Comparisons of m^* and m^o for different K.

Fig. 5 compares m^* with m^o to show the accuracy of prediction of m^*. We fix channel AOD and AOA of dominant path while $g_1 \sim \mathcal{CN}(0,1)$ is randomly generated with 10^4 implementations. It is shown that in general m^* can well approach m^o. With the increasing K, the optimal level achieving the weighted sum-rate maximization tends to be higher level. If K is sufficiently large, $(K - K_t)/K \approx 1$, implying that the weighted sum-rate is simplified to sum-rate where the search of the codebook will always stop at the highest level. Given K, it is seen that in high SNR region, e.g., $\gamma = 20$dB, the optimal level m^o tends to be small; while in lower SNR region, e.g., $\gamma = 0$dB, the optimal level m^o tends to be large. The reason is that the receiving SNR is enough when γ is large, it is better to stop the search at lower level to save the training time K_t so that the weighted sum-rate can be improved. However, when γ is small, we have to search high level codewords with high directional gain to compensate the small γ. Also note that the gap between m^* and m^o is relatively larger in transitional region of different levels than the gap in the other regions, which is caused by the discontinuity of directional gain at different levels.

VI. Conclusions

In this paper we have studied the weighted sum-rate maximization for mmWave massive MIMO communications by jointly considering the duration for channel training and the receiving SNR. We have proposed an algorithm to design the analog beamforming and analog combining based on the hierarchical codebook. Simulation results show that the proposed algorithm outperforms multi-sectional search, where the former can achieve the weighted sum-rate almost twice of the latter at certain length of transmission block when the channel SNR is 20dB.

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