Resource Efficiency: A New Beamforming Design for Multicell Multiuser Systems

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Abstract—Apart from spectral efficiency (SE), energy efficiency (EE) has become an important metric for future wireless communication systems. However, how to achieve balance between SE and EE for a multicell multiuser coordinated beamforming system remains an open problem. In this paper, we propose a new paradigm for EE-SE balancing, namely, using a new metric called resource efficiency (RE) as the objective function to achieve a reasonable tradeoff between SE and EE. The considered optimization problem is more complex than traditional problems and, in general, is NP-hard for finding the globally optimal solution. To obtain insight into the problem, we first equivalently transform the original problem into a parameterized subtractive form optimization problem with some auxiliary variables by using the fraction programming theory. Then, a successive convex approximation method and the classical second-order cone programming method are jointly used to solve the equivalent problem. Finally, an effective algorithm with proven convergence is proposed to obtain the solution to the considered RE optimization. In addition, the algorithm is further extended to consider imperfect channel state information (CSI) using the worst-case design. Extensive numerical simulations are given to validate the effectiveness of our algorithm.

Index Terms—Coordinated beamforming, geometry programming, resource efficiency (RE), second-order cone programming (SOCP), successive convex approximation.

I. INTRODUCTION

I N recent years, the exponentially growing data traffic and the requirement of ubiquitous access have triggered dramatic densification and expansion of network infrastructures. As a consequence, the energy crisis and global warming problems have emerged with the drastic increase in infrastructure nodes and have attracted much attention and lead to an urgent

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need for much research on energy efficiency (EE) [1], [2]. In addition, fifth-generation cellular communication systems are expected to significantly improve total system capacity, as well as EE, compared with fourth-generation systems. Motivated by this, the GreenTouch Constrium has been founded and aims to improve the end-to-end EE by 1000-fold. In particular, Green Transmission Technology, which is one of the biggest umbrella projects in GreenTouch, has been provided a grant to focus on the energy-efficient design of physical-layer transmission technologies and medium-access-control-layer radio resource management in wireless networks [3].

Energy-efficient transmission has been extensively investigated in the literature with a goal of improving the EE of wireless communication networks [4]-[9]. Energy-efficient orthogonal frequency-division multiple access (OFDMA) was first devised in [4], showing that EE optimization promises at least a 20% reduction in power consumption. Energy-efficient resource allocation was also investigated in coordinated multicell OFDMA downlink systems with limited backhaul [5] and was further studied with a hybrid energy harvesting base station (BS) [6]. On the other hand, coordinated energy-efficient beamforming design for a multicell multiuser downlink system has also attracted much attention. In [7], a new beamforming method was proposed to maximize the network EE. In [8], the design of multicell multiuser precoding was further investigated to optimize the weighted sum EE by taking into account the heterogeneity of future cellular networks. More recently, energy-efficient resource allocation was also devised for robust cognitive radio wireless networks [9].

Although EE has become an important metric in nextgeneration cellular systems, we have to mention that spectral efficiency (SE) always plays a central role in the system design. As a result, the weighted sum-rate maximization (WSRMax) problem for multicell multiuser beamforming has been intensively studied in recent years; however, this remains an open problem due to its nonconvexity [10]. To address this issue, various numerical methods have been developed in the literature [11]–[14]. In [11], an effective algorithm based on successive convex approximation was developed to solve the WSRMax problem and provided a significant performance improvement over the existing iterative waterfilling algorithm. In [12], a centralized and exponentially complex global optimization approach was proposed to reach an exact solution via the branchand-bound method. To reduce the computational complexity, an interesting distributed algorithm for the WSRMax problem was devised by exploiting the equivalence between the WSRMax problem and a weighted sum mean-square-error minimization

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Unfortunately, previous results reveal that the separate optimization of SE and EE dose not always coincide. As shown in [4]–[9], in the low signal-to-noise ratio (SNR) regime, the optimal SE and EE are achieved at the same time by transmitting at full power, whereas they conflict in the middle-to-high-SNR regime. Therefore, how to balance SE and EE becomes an important problem [15]. The conventional design, which mainly focuses on maximizing SE or EE separately, cannot achieve this goal. Recently, the tradeoff between SE and EE has been investigated by formulating the EE maximization problem subject to given SE requirements. In [16], a tight upper bound and a tight lower bound on the SE-EE curve for a downlink OFDMA network were obtained, which reflected the actual SE-EE relation. In [17], the tradeoff between SE and EE was investigated for an interference-limited wireless network. In [18], an uplink-downlink duality-based beamforming approach was proposed to achieve a reasonable SE and EE tradeoff in a multiuser downlink system. In addition, the relation between SE and EE was further investigated for the large-scale multiuser multiple-input multiple-output (MIMO) system with fixed transceiver beamformers [19]. To further balance the SE and the EE, a new paradigm for the SE and EE tradeoff was proposed in [20], namely, a new evaluation metric of resource efficiency (RE) was used as the objective function to optimize the resource allocation of a single-cell downlink OFDMA system. It was proved that the proposed RE is capable of exploiting the tradeoff between SE and EE by balancing consumption power and occupied bandwidth. However, this work only focused on power allocation optimization without taking intercell and interuser interference into account.

Rather than focusing on either SE or EE separately, recent works have studied the relationship between EE and SE, providing good insight into the achievable tradeoff between SE and EE. However, how to obtain balance between SE and EE for a multicell multiuser coordinated beamforming system remains an open problem. In this paper, we study the SE and EE balancing problem for the multicell mulliuser coordinated beamforming systems. The problem of interest is formulated as maximizing the RE subject to per-user quality of service (QoS) requirements and per-BS power constraints. Note that different from the single-cell OFDMA power allocation optimization problem considered in [20], our coordinated beamforming optimization needs to tackle both the interuser interference and intercell interference for multicell multiuser interference networks. On the other hand, contrary to the conventional WSRMax problem and the EE maximization problem, the considered RE optimization problem is more complex and, in general, NP-hard for finding the globally optimal solution, which is due to the fact that the RE objective acts as a sum of two nonconvex functions. Furthermore, conventional convex optimization tools cannot be used directly to solve the problem of interest. To obtain insight into the problem, the original optimization objective function is first transformed into a standard fraction programming optimization function with a ratio of concave function to convex function form. A parameterized subtractive form optimization problem is further obtained based on the fraction programming theory [8] by introducing some auxiliary variables. Then, an effective algorithm with proven convergence is proposed to obtain a solution to the considered RE optimization by jointly exploiting the successive convex approximation for low complexity (SCALE) method and the classical second-order cone programming (SOCP) method. In addition, the algorithm is further extended to consider imperfect channel state information (CSI) using the worst-case design. Finally, extensive numerical simulations are given to evaluate the performance of our algorithm.

This rest of this paper is organized as follows. The system model is described in Section II. In Section III, a resourceefficient beamforming algorithm is proposed for the downlink of a multicell multiuser system subject to per-BS power constraints and individual QoS requirements. The simulation results are shown in Section IV, and conclusions are finally given in Section V.

The following notations are used throughout this paper. Bold lowercase and uppercase letters represent column vectors and matrices, respectively. The superscript T , H , *, and † represent the transpose operator, the conjugate transpose operator, the conjugate operator, and the Moore–Penrose pseudoinverse of a matrix, respectively. $\|\cdot\|$ denotes the ℓ_2 -norm.

II. SYSTEM MODEL

Consider the downlink of a multicell multiuser network consisting of K cells. Within each cell j, there is a BS equipped with M_j transmit antennas and N_j single-antenna users. Let user (j, k) and BS j denote, respectively, the kth user and the BS in cell j, j = 1, ..., K. Let $h_{m,j,k} \in \mathbb{C}^{M_m \times 1}$ denote the flat-fading channel coefficient from BS m to user (j, k), which includes large-scale fading, small-scale fading, and shadow fading.¹ Let $w_{j,k} \in \mathbb{C}^{M_m \times 1}$ denote the normalized transmit beamformer that BS j uses to transmit a single stream of data signal $x_{j,k}$ to user (j, k) with $\mathbb{E}\{x_{j,k}\} = 0$ and $\mathbb{E}\{|x_{j,k}|^2\} = 1$. The received signal of user (j, k) is calculated as

$$y_{j,k} = \sum_{m=1}^{K} \boldsymbol{h}_{m,j,k}^{H} \sum_{n=1}^{N_{m}} \sqrt{p_{m,n}} \boldsymbol{w}_{m,n} x_{m,n} + z_{j,k} \qquad (1)$$

where $p_{m,n}$ denotes the transmit power for user (m, n), and $z_{j,k}$ is a zero-mean circularly symmetric complex Gaussian random noise with power spectrum density $\sigma_{j,k}^2$. We further assume that the signals for different users are independent from each other and from the receiver noise. The instantaneous rate of user (j, k) is calculated as

$$R_{j,k} = \mathcal{W}\log_2(1 + \mathrm{SINR}_{j,k}) \tag{2}$$

where W represents the occupied bandwidth, and $SINR_{j,k}$ denotes the signal-to-interference-plus-noise ratio of user (j, k)

¹For a time-division duplex system, the transmitters can estimate the channels from the sounding signals received in the reverse link. For a frequencydivision duplex system, the transmitters are provided with quantized CSI via feedback.

and is given by

$$\operatorname{SINR}_{j,k} = \frac{p_{j,k} \left| \boldsymbol{h}_{j,j,k}^{H} \boldsymbol{w}_{j,k} \right|^{2}}{\sum_{(m,n) \neq (j,k)} p_{m,n} \left| \boldsymbol{h}_{m,j,k}^{H} \boldsymbol{w}_{m,n} \right|^{2} + \mathcal{W} \sigma_{j,k}^{2}}.$$
 (3)

For notational convenience, let $W_j = \{w_{j,1}, \dots, w_{j,N_j}\}$ and $p_j = \{p_{j,1}, \dots, p_{j,N_j}\}$ denote, respectively, the multiuser precoder set and power allocation set of BS j, and let W = $\{W_1, \dots, W_K\}$ and $p = \{p_1, \dots, p_K\}$ denote the collection of all precoders and the collection of all power allocation vectors, respectively. In the existing literature, one mainly focuses on maximizing separately the SE or the EE under some predefined constraints for wireless communication systems. Often, the weighted sum per-cell EE maximization (WSEEMax) problem subject to per-user QoS constraints and per-BS power constraints is formulated as follows [8]:

$$\max_{\boldsymbol{W},\boldsymbol{p}} \sum_{j=1}^{K} \frac{\kappa_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} R_{j,k}}{\xi \sum_{k=1}^{N_{j}} p_{j,k} + M_{j} P_{c} + P_{0}}$$

s.t. SINR_{j,k} $\geq \gamma_{j,k}, \|\boldsymbol{w}_{j,k}\| = 1, \sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j} \quad \forall j, k.$ (4)

On the other hand, the traditional WSRMax problem subject to some given QoS demands and per-BS transmit power constraints is defined as

$$\max_{\boldsymbol{W},\boldsymbol{p}} \sum_{j=1}^{K} \sum_{k=1}^{N_j} \alpha_{j,k} R_{j,k}$$

s.t. SINR_{j,k} $\geq \gamma_{j,k}, \|\boldsymbol{w}_{j,k}\| = 1, \sum_{k=1}^{N_j} p_{j,k} \leq P_j \quad \forall j,k.$ (5)

Note that the aforementioned optimization problems focus on either SE or EE separately without considering the tradeoff between them. Motivated by this observation, in this paper, our goal is to achieve a tunable tradeoff between SE and EE. In particular, the optimization problem of interest combines SE and EE together and is mathematically formulated as follows:

$$\max_{\boldsymbol{W},\boldsymbol{p}} \sum_{j=1}^{K} \frac{\kappa_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} R_{j,k}}{\xi \sum_{k=1}^{N_{j}} p_{j,k} + M_{j} P_{c} + P_{0}} + \sum_{j=1}^{K} \frac{\beta_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} R_{j,k}}{\xi P_{j} + M_{j} P_{c} + P_{0}}$$

s.t. SINR_{j,k} $\geq \gamma_{j,k}, \sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j}, \|\boldsymbol{w}_{j,k}\| = 1 \quad \forall j,k$ (6)

where the weight $\alpha_{j,k}$ is used to represent the priority of user (j,k) in the system; κ_j denotes the priority of the EE in cell $j; \xi \geq 1$ is a constant that accounts for the inefficiency of the power amplifier; P_c is the constant circuit power consumption per antenna; P_0 is the basic power consumed at the BS, which is independent of the number of transmit antennas [5]–[9]; P_j is the transmit power constraint of BS $j; \gamma_{j,k}$ is the target SINR of user (j,k); and β_j is a weight factor used to tune the achievable tradeoff between EE and SE. In addition, it is easy to see that

RE optimizes EE when $\beta_j = 0$ and optimizes SE when $\beta_j = \infty$. Note that there is no *a priori* correspondence between the weight vector $\beta = \{\beta_1, \dots, \beta_K\}$ and a beamforming solution; hence, it is up to the decision maker to choose an appropriate weight vector. Without loss of generality, we consider β as a constant in our RE optimization problem [20].

The main difference between the considered problem in [20] and the RE optimization problem of interest is that the former only aims to optimize the power allocation for singlecell OFDMA dowlink systems where the user rate function is convex, whereas the objective of our interest is to maximize the RE by simultaneously optimizing the transmit power vector and beamforming vectors for the downlink of a multicell multiuser interference network. Furthermore, the results in [7]-[13] have shown that for interference channel communication networks, problems (4) and (5) are both nonconvex and, therefore, are difficult to achieve the globally optimal solution due to the coupling between optimization variables. Hence, convex optimization tools cannot be directly used to solve them. It is worth mentioning that, in contrast to the conventional WSRMax maximization problem and the EE optimization problem, the RE problem under consideration has more drawbacks, as its objective function is a weighted sum of the system EE and the system SE. In particular, the nonconvex nature of the quadratic constraints $SINR_{i,k}$, the fractional form of the EE definition, and the user rate cause problem (6) to be nonconvex, and it is very difficult to obtain the globally optimal solution [21]–[23].

III. RESOURCE-EFFICIENT BEAMFORMING ALGORITHM DESIGN

Here, we jointly exploit the SCLAE method and the fractional programming method to design an effective algorithm for the optimization problem (6) with the aim to maximize the weighted sum SE and EE. To obtain a concave–convex form of the fractional objective function, we introduce auxiliary variables $\eta_{j,k}$ for user $(j,k) \forall j, k$, to reformulate problem (7), shown below, into the following equivalent form:

$$\max_{\boldsymbol{W},\boldsymbol{p},\boldsymbol{\eta}} \sum_{j=1}^{K} \left(\frac{\kappa_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} \overline{R}_{j,k}}{\xi \sum_{k=1}^{N_{j}} p_{j,k} + M_{j} P_{c} + P_{0}} + \frac{\beta_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} \overline{R}_{j,k}}{\xi P_{j} + M_{j} P_{c} + P_{0}} \right)$$

s.t. SINR_{j,k} $\geq \gamma_{j,k}$, SINR_{j,k} $\geq \eta_{j,k}$, $\sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j} \quad \forall j$ (7)

where $\overline{R}_{j,k} = \ln(1 + \eta_{j,k}) \forall j, k$, and η denotes the collection of all auxiliary variables $\eta_{j,k} \forall j, k$. Note that in the given equivalent problem, a constant factor $W/\ln(2)$ in the objective function is omitted without impacting the final solution. It is easily proved, by contradiction, that in the optimal solution to problem (7), the inequalities SINR_{*j*,*k*} $\geq \eta_{j,k} \forall j, k$, all meet with equality, i.e., SINR_{*j*,*k*} = $\eta_{j,k} \forall j, k$. Although the objective function of problem (7) has a simple form compared with that of problem (6), it is still a nonconvex problem due to the nonconvex nature of the first item in the objective function and the QoS constraints SINR_{*j*,*k*} $\geq \eta_{j,k} \forall j, k$. One can see that problem (7) belongs to a class of sum-ofratio optimization problems that seek to minimize/maximize the sum of fractional functions and has been actively studied for several decades [8], [24]. Instead of directly solving problem (7), we resort to obtaining a tractable form and reformulating it into the following form by introducing auxiliary variables $\mu = {\mu_1, \ldots, \mu_K}$:

$$\max_{\boldsymbol{W},\boldsymbol{p},\boldsymbol{\eta}} \sum_{j=1}^{K} \left(\kappa_{j} \mu_{j} + \frac{\beta_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} \overline{R}_{j,k}}{\xi P_{j} + M_{j} P_{c} + P_{0}} \right)$$

s.t. SINR_{j,k} $\geq \gamma_{j,k}$, SINR_{j,k} $\geq \eta_{j,k}$, $\sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j} \quad \forall j$
 $\sum_{k=1}^{N_{j}} \alpha_{j,k} \overline{R}_{j,k} \geq \mu_{j} \left(\xi \sum_{k=1}^{N_{j}} p_{j,k} + M_{j} P_{c} + P_{0} \right) \quad \forall j.$
(8)

It is easy to prove that the last K inequalities named EE constraints meet with equality when the optimal solution of problem (8) is obtained. To obtain more insight into problem (8), we derive the following proposition.

Proposition 1: If $(\overline{W}, \overline{p}, \overline{\eta}, \overline{\mu})$ is the solution of problem (8), then there exist $\overline{\lambda}$ explained as the Lagrange multipliers associated with EE constraints, such that $(\overline{W}, \overline{p}, \overline{\eta})$ satisfies the Karush–Kuhn–Tucker (KKT) conditions of the following problem for $\lambda = \overline{\lambda}$ and $\mu = \overline{\mu}$:

$$\max_{\boldsymbol{W},\boldsymbol{p},\boldsymbol{\eta}} \sum_{j=1}^{K} \left(f_{j}(\boldsymbol{p},\boldsymbol{\eta}) + \frac{\beta_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} \overline{R}_{j,k}}{\xi P_{j} + M_{j} P_{c} + P_{0}} \right)$$

s.t. SINR_{j,k} $\geq \gamma_{j,k}$, SINR_{j,k} $\geq \eta_{j,k}$, $\sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j} \forall j$ (9)

where

$$f_j(\boldsymbol{p}, \boldsymbol{\eta}) = \lambda_j \left(\sum_{k=1}^{N_j} (\alpha_{j,k} \overline{R}_{j,k} - \mu_j \xi p_{j,k}) - \mu_j (M_j P_c + P_0) \right)$$

and $(\overline{W}, \overline{p}, \overline{\eta})$ also satisfies the following system equations for $\lambda = \overline{\lambda}$ and $\mu = \overline{\mu}$:

$$\begin{cases} \lambda_j = \frac{\kappa_j}{\underset{k=1}{\overset{N_j}{\sum} p_{j,k} + M_j P_c + P_0}} \quad \forall j \tag{10a} \end{cases}$$

$$\mu_{j} = \frac{\sum\limits_{k=1}^{N} \alpha_{j,k} \overline{R}_{j,k}}{\sum\limits_{k=1}^{N_{j}} p_{j,k} + M_{j} P_{c} + P_{0}} \quad \forall j.$$
 (10b)

On the contrary, if $(\overline{W}, \overline{p}, \overline{\eta})$ is a solution of problem (9) and satisfies system equation (10) for $\lambda = \overline{\lambda}$ and $\mu = \overline{\mu}$,

 $(\overline{W}, \overline{p}, \overline{\eta}, \overline{\mu})$ also satisfies the KKT conditions of problem (8) for Lagrange multipliers $\lambda = \overline{\lambda}$ associated with EE constraints.

Proof: The proof of Proposition 1 is similar to the proof of Theorem 1 [8, p. 744]. Therefore, the detailed proof of Proposition 1 is omitted.

Compared with the fractional form objective function in problem (7), although problem (9) has a subtractive form objective function by introducing some auxiliary variables whose solution can be obtained via an iterative optimization algorithm [8], it is still nonconvex due to the QoS constraints SINR_{*j*,*k*} $\geq \eta_{j,k} \forall j, k$. In what follows, we first design an effective algorithm to solve problem (9) for fixed values of auxiliary variables λ and μ . To obtain the solution to problem (9) with respect to variable W, p, η , the block-coordinated monotonic optimization method is then adopted to solve the RE optimization problem (9). More specifically, we maximize the objective value of problem (9) by sequentially fixing two of the three variables W, p, η and updating the third.

A. Power Allocation Optimization

Here, we focus on the optimization of the power allocation for the considered RE optimization problem. Note that for fixed auxiliary variables λ, μ , and beamforming vector W^* , problem (9) is a signomial optimization problem and is nonconvex in general. In addition, the nonconvex nature of constraints SINR_{*j*,*k*} $\geq \eta_{j,k} \forall j, k$, makes the considered optimization problem more complex and hard to find the globally optimal solution. Fortunately, note that all constraints are a monomial function in problem (9). Therefore, if the objective function can be reformulated as a monomial function, problem (9) becomes a geometric programming (GP) problem in standard form. The GP problem can be reformulated as convex problems, and they can be solved very efficiently, even for large-scale problems [28]. To obtain further a tractable convex formulation and design an efficient solution of problem (9), next, we make use of the lower bound $\ln(1+z) \ge -(\phi/z) + \varphi$ that is tight at $z = z_0$ when the approximation constants are chosen as [11], [25]

$$\phi = \frac{z_0^2}{1+z_0}, \varphi = \frac{z_0}{1+z_0} + \ln(1+z_0).$$
(11)

In the sequel, instead of solving problem (9) directly, we resort to solving the following lower-bound-approximated optimization problem:

$$\max_{\boldsymbol{p},\boldsymbol{\eta}} \sum_{j=1}^{K} \left(\widetilde{f}_{j}(\boldsymbol{p},\boldsymbol{\eta}) + \frac{\beta_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} \widetilde{R}_{j,k}}{\xi P_{j} + M_{j} P_{c} + P_{0}} \right)$$

s.t. SINR_{j,k} $\geq \gamma_{j,k}$, SINR_{j,k} $\geq \eta_{j,k}$, $\sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j} \quad \forall j,k$
 $(1 - \omega_{j,k}) \overline{\eta}_{j,k} \leq \eta_{j,k} \leq (1 + \omega_{j,k}) \overline{\eta}_{j,k} \quad \forall j,k \quad (12)$

where $\widetilde{R}_{j,k} = -(\phi_{j,k}/\eta_{j,k}) + \varphi_{j,k}$, $\widetilde{f}_j(\boldsymbol{p}, \boldsymbol{\eta}) = \lambda_j (\sum_{k=1}^{N_j} \alpha_{j,k})$ $\widetilde{R}_{j,k} - \mu_j (\xi \sum_{k=1}^{N_j} p_{j,k} + M_j P_c + P_0))$, $\phi_{j,k}$, and $\varphi_{j,k}$ are approximation constants at approximated point $\overline{\eta}_{i,k}$, given by

$$\phi_{j,k} = \frac{\overline{\eta}_{j,k}^2}{1 + \overline{\eta}_{j,k}}, \ \varphi_{j,k} = \frac{\overline{\eta}_{j,k}}{1 + \overline{\eta}_{j,k}} + \ln(1 + \overline{\eta}_{j,k})$$
(13)

and $\{\omega_{j,k}\} > 1$ are parameters used to control the desired approximation accuracy. The last set of inequality constraints of problem (12) is called trust region constraints and restricts the range of variables η to a region where the monomial approximation is accurate enough [26]. In addition, problem (12) can be further reformulated into problem (14), shown at the bottom of the page, in which some constant items are removed without affecting the final solutions, where the approximation vectors ϕ and φ are fixed. Although the QoS constraints SINR_{*j*,*k*} $\geq \eta_{j,k}$, $\forall j, k$, are all nonconvex, the objective function and the constraint conditions fortunately are all posynomial functions with respect to the optimization variables p and η . Based on these observations, the following result gives a way forward to address the nonconvex nature of QoS constraints SINR_{*j*,*k*} $\geq \eta_{j,k}$, $\forall j, k$ [27].

Lemma 1: Problem (14) can be reformulated into a convex form by the transformation $\tilde{p}_{i,k} = \ln(p_{i,k})$ and $\tilde{\eta}_{i,k} = \ln(\eta_{i,k})$.

Lemma 1 ensures that problem (14) can be easily transformed into the classical GP problem. Furthermore, it is well known that a GP problem can be efficiently solved using a standard convex optimization approach [28]. These observations suggest that, by starting from an initial point, a close local optimum solution can be obtained by solving a sequence of GPs that locally approximate the original problem. Thus, an iterative algorithm consisting of the maximization step (M-step) that is used to update p and η for fixed beamforming vector W^* , ϕ , and φ , and the tighten step (T-step) that is used to update the value of ϕ , and φ , can be developed to solve problem (14) with fixed beamforming vector W^* [25]. A very brief outline of the proposed successive approximation algorithm, which is used to update p and η , is summarized as Algorithm 1.

Algorithm 1 Power Allocation Optimization Algorithm

- 1: Let Ite = 0, initialize the value of $\overline{\eta}^{(Ite)}$, compute $\phi^{(Ite)}$ and $\psi^{(Ite)}$ based on (13);
- 2: (M-step): Solve problem (14) with $\phi^{(Ite)}$, $\psi^{(Ite)}$ for fixed beamforming vector W^* by using GP algorithm, and obtain $p^{(Ite+1)}$ and $\eta^{(Ite+1)}$;
- 3: (T-step): Let $\overline{\eta}^{(Ite+1)} = \eta^{(Ite+1)}$, then update ϕ and ψ with $\overline{\eta}^{(Ite+1)}$ via (10), and obtain $\phi^{(Ite+1)}$ and $\psi^{(Ite+1)}$;

4: Let Ite = Ite + 1, repeat step 2 and step 3 until convergence.

Remark 1: The parameters $\{\omega_{j,k}\}$ control the desired approximation accuracy. However, they also influence the convergence speed of Algorithm 1. At every step, each entry of the current SINR guess $\overline{\eta}_{j,k}$ can be increased or decreased, at most, by a factor $\{\omega_{j,k}\}, \forall j, k$. In other words, good accuracy for the monomial approximations can be achieved when the values of $\{\omega_{j,k}\}$ are close to 1 at the expense of slower convergence speed; on the other hand, the convergence speed can be speeded up when the values of $\{\omega_{j,k}\}$ are larger than 1, at the cost of reduced accuracy. In most practical cases, a fixed value $\{\omega_{j,k} = 0.1\}$ offers a good tradeoff between speed and accuracy [28], [29].

Lemma 2: The sequence generated in the iteration of Algorithm 1 produces a monotonically increasing objective of problem (13) for fixed beamforming vector W^* and always converges.

Proof: The proof of Lemma 2 is similar to the proof of [25, Th. 2, p. 26]. Hence, the detailed proof is omitted.

B. Beamformer Optimization

For ease of notation, let p^* and η^* be the optimal solution to problem (14) with fixed λ , μ , and W^* via the SCALE method. Recalling the constraints in problem (12), it is easy to see that we have $\eta_{j,k}^* \ge \gamma_{j,k} \forall j, k$. It is easy to see that the RE and EE can be further improved while maintaining the SE by minimizing the transmit power according to the formulation of problem (9). Motivated by this observation, before proceeding to update the transmit beamforming vector W, we resort to solving a weighted sum power minimization problem, which is described as²

$$\min_{\{\widetilde{\boldsymbol{w}}_{j,k}\}} \sum_{j=1}^{K} \sum_{k=1}^{N_j} \lambda_j \mu_j \|\widetilde{\boldsymbol{w}}_{j,k}\|^2$$

s.t. $\overline{\text{SINR}}_{j,k} \ge \eta_{j,k}^*, \sum_{k=1}^{N_j} \|\widetilde{\boldsymbol{w}}_{j,k}\|^2 \le P_j \quad \forall j,k$ (15)

where $\overline{\text{SINR}}_{j,k}$ is defined as

$$\overline{\text{SINR}}_{j,k} = \frac{\left|\boldsymbol{h}_{j,j,k}^{H} \widetilde{\boldsymbol{w}}_{j,k}\right|^{2}}{\sum_{(m,n) \neq (j,k)} \left|\boldsymbol{h}_{m,j,k}^{H} \widetilde{\boldsymbol{w}}_{m,n}\right|^{2} + \mathcal{W}\sigma_{j,k}^{2}}.$$
 (16)

Compared with the conventional sum power minimization problem, the sum power minimization problem of interest is

²Note that the sum power minimization problem aims to use minimum transmit power to realize the same system sum rate as achieved by the solution of problem (12) with fixed beamformers. In addition, the sum power minimization problem can also be used to check the feasibility of the target SINR requirements γ and initialize the transmit power vector p and the transmit beamforming vector W by setting $\lambda_i = \mu_i = 1 \forall j$ in problem (15).

$$\min_{\boldsymbol{p},\boldsymbol{\eta}} \sum_{j=1}^{K} \sum_{k=1}^{N_{j}} \left(\left(\lambda_{j} \alpha_{j,k} \phi_{j,k} + \frac{\beta_{j} \alpha_{j,k} \phi_{j,k}}{\xi P_{j} + M_{j} P_{c} + P_{0}} \right) \eta_{j,k}^{-1} + \lambda_{j} \mu_{j} \xi p_{j,k} \right)$$
s.t. SINR_{j,k} $\geq \gamma_{j,k}$, SINR_{j,k} $\geq \eta_{j,k}$, $\sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j}, (1 - \omega_{j,k}) \overline{\eta}_{j,k} \leq \eta_{j,k} \leq (1 + \omega_{j,k}) \overline{\eta}_{j,k} \quad \forall j,k$ (14)

weighted by the parameters of the fractional programming λ_j and μ_j for cell $j \forall j$. It is easy to know that problem (15) can be solved via the classical SOCP optimization tools [30]. For simplicity, let \overline{W}^* be the optimal solution to problem (15) with given target SINR η^* . It is easily seen that $\eta_{j,k}^* \ge \gamma_{j,k} \forall j, k$, according to the solution to problem (7), and the following Lemma 3 can be easily obtained.

Lemma 3: The RE achieved by the beamformers \overline{W}^* is not less than that achieved by W^* and p^* .

Proof: The optimal solution \overline{W}^* to problem (15) aims to minimize the transmit power with the target user SINR achieved by W^* and p^* as the target SINR η^* .³ According to the constraints in problem (14), it is easy to have the relation of $\eta_{j,k}^* \ge \gamma_{j,k}$, $\forall j, k$. Otherwise, we can obtain a larger value of the objective function by letting $\eta_{j,k}^* = \gamma_{j,k} \forall j, k$, which is a contradiction.

In addition, it is also easy to prove $\sum_{j=1}^{K} \sum_{k=1}^{N_j} \lambda_j \mu_j$ $\|\overline{w}_{j,k}\|^2 \leq \sum_{j=1}^{K} \sum_{k=1}^{N_j} \lambda_j \mu_j p_{j,k}^*$. Otherwise, the total transmit power can be further reduced by letting $\overline{w}_{j,k} = \sqrt{p_{j,k}^*} w_{j,k}^* \forall j, k$, which is a contradiction. It also means that \overline{W}^* achieves the same user rate, which is achieved by W^* and p^* without increasing transmit power consumption. In other words, $\overline{\text{SINR}}_{j,k} = \eta_{j,k}^*$ and $\overline{\text{SINR}}_{j,k} = \eta_{j,k}^* \forall j, k$, all hold, i.e., it also means that the EE and RE achieved by \overline{W}^* are no less than that achieved by W^* , and p^* results in an increasing objective value of problem (7).

C. RE Optimization Algorithm

Based on the given analysis, a two-layer alternating optimization algorithm is proposed to solve problem (7). In the outer layer, the auxiliary variables λ and μ are updated using a Newton-like method. In the inner layer, the variables W and p are updated through (8). The detailed steps are summarized in Algorithm 2, where $\psi_j(\lambda_j)$, $\varphi_j(\mu_j)$, and χ_j are defined, respectively, by (17), shown at the bottom of the page, and ρ denotes the objective value of (9) for fixed λ and μ . The major computational complexity of the developed algorithm arises from the beamforming vector update step, i.e., the classical

³If we let $\boldsymbol{w}_{j,k}^* = \overline{\boldsymbol{w}}_{j,k}^* / \|\overline{\boldsymbol{w}}_{j,k}^*\|$ and $p_{j,k} = \|\overline{\boldsymbol{w}}_{j,k}^*\|^2$, recalling $\sum_{k=1}^{N_j} \|\overline{\boldsymbol{w}}_{j,k}^*\|^2 \leq P_j \forall j$, we have $\sum_{k=1}^{N_j} p_{j,k} \leq P_j$, $\forall j$. According to the proof of Lemma 3, we have $\overline{\text{SINR}}_{j,k} = \eta_{j,k}^*$, $\overline{\text{SINR}}_{j,k} = \eta_{j,k}^* \geq \gamma_{j,k}$, and $(1 - \omega_{j,k})\overline{\eta}_{j,k} \leq \eta_{j,k}^* \leq (1 + \omega_{j,k})\overline{\eta}_{j,k} \forall j, k$. Thus, it is easy to see that $\overline{\boldsymbol{W}}^*$ is also the solution of problem (14).

SOCP programming algorithm whose computational complexity depends on the number of the variables and the constraint equations of the corresponding optimization problem [30], [32].

Algorithm 2 RE Optimization Algorithm

1: Choose $\forall \xi \in (0,1) \ \forall \varepsilon \in (0,1)$ and choose arbitrarily $W^{(*)}$ and $p^{(*)}$ such that it satisfies the power constraints. Let $\rho^{(*)} = 0$, and

$$\int_{\lambda_{j}^{(*)}} \lambda_{j}^{(*)} = \frac{\kappa_{j}}{\xi \sum_{\substack{k=1\\N_{j}}}^{N_{j}} p_{j,k}^{(*)} + M_{j} P_{c} + P_{0}} \quad \forall j$$
(18a)

$$\begin{pmatrix}
\mu_{j}^{(*)} = \frac{\sum_{k=1}^{N_{j}} \alpha_{j,k} \overline{R}_{j,k}}{\xi \sum_{k=1}^{N_{j}} p_{j,k}^{(*)} + M_{j} P_{c} + P_{0}} \quad \forall j
\end{cases}$$
(18b)

- 2: Update p and η by solving problem (14) with Algorithm 1 for fixed W^* , $\lambda^{(*)}$, and $\mu^{(*)}$, then obtain $p^{(**)}$ and $\eta^{(**)}$. Solve problem (15) with $\eta^{(**)}$, and obtain $\overline{W}^{(**)}$. Let $p_{j,k}^{(**)} = \|\overline{w}_{j,k}^{(**)}\|^2$ and $w_{j,k}^{(**)} = \overline{w}_{j,k}^{(*)} / \|\overline{w}_{j,k}^{(*)}\|$, $\forall j, k$. Calculate the value of ρ with $p^{(**)}$ and $W^{(**)}$, and obtain $\rho^{(**)}$.
- 3: If $|\rho^{(**)} \rho^{(*)}| \le \varsigma$, where ς is a predefined threshold, then let $W^{(*)} = W^{(**)}$, $p^{(*)} = p^{(**)}$, $\rho^{(*)} = \rho^{(**)}$ and go to step 4. Otherwise, let $W^{(*)} = W^{(**)}$, $p^{(*)} = p^{(**)}$, $\rho^{(*)} = \rho^{(**)}$, and go to step 2.
- 4: If the following conditions are satisfied:

$$\begin{cases} \lambda_{j}^{(*)} \left(\xi \sum_{k=1}^{N_{j}} p_{j,k}^{(*)} + M_{j}P_{c} + P_{0} \right) - \kappa_{j} = 0 \ \forall j \qquad (19a) \\ \mu_{j} \left(\xi \sum_{k=1}^{N_{j}} p_{j,k} + M_{j}P_{c} + P_{0} \right) - \sum_{k=1}^{N_{j}} \alpha_{j,k} \overline{R}_{j,k} = 0 \ \forall j \qquad (19b) \end{cases}$$

then output the optimal solutions $W^{(*)}$ and $p^{(*)}$, and stop the algorithm. Otherwise, let $i^{(*)}$ denote the smallest integer among $i \in \{0, 1, 2, \dots\}$ satisfying

$$\sum_{j=1}^{K} \left| \psi_j \left(\lambda_j^{(*)} - \xi^i \chi_j \psi_j \left(\lambda_j^{(*)} \right) \right) \right|^2$$

+
$$\sum_{j=1}^{K} \left| \varphi_j \left(\mu_j^{(*)} - \xi^i \chi_j \varphi_j \left(\mu_j^{(*)} \right) \right) \right|^2 \le \left(1 - \varepsilon \xi^i \right)^2$$

$$\times \sum_{j=1}^{K} \left(\left| \psi_j \left(\lambda_j^{(*)} \right) \right|^2 + \left| \varphi_j \left(\mu_j^{(*)} \right) \right|^2 \right)$$
(20)

$$\left(\psi_j(\lambda_j) = \lambda_j \left(\xi \sum_{k=1}^{N_j} p_{j,k} + M_j P_c + P_0\right) - \kappa_j \quad \forall j$$
(17a)

$$\begin{cases} \varphi_j(\mu_j) = \mu_j \left(\xi \sum_{k=1}^{N_j} p_{j,k} + M_j P_c + P_0 \right) - \sum_{k=1}^{N_j} \alpha_{j,k} \overline{R}_{j,k} \quad \forall j \end{cases}$$
(17b)

$$\chi_{j} = \frac{1}{\xi \sum_{k=1}^{N_{j}} p_{j,k} + M_{j} P_{c} + P_{0}}, \quad \forall j$$
(17c)

with $\boldsymbol{W}^{(*)}$ and $\boldsymbol{p}^{(*)}$, then

$$\begin{cases} \lambda_{j}^{(**)} = \lambda_{j}^{(*)} - \xi^{i^{(*)}} \chi_{j} \psi_{j} \left(\lambda_{j}^{(*)} \right) & \forall j \\ \gamma_{j}^{(**)} = \gamma_{j}^{(*)} \gamma_{j}^{(*)} \left(\gamma_{j}^{(*)} \right) & \forall j \end{cases}$$
(21a)

$$\left(\beta_{j}^{*}\right)^{\prime} = \beta_{j}^{*} - \xi^{*} \chi_{j} \varphi_{j} \left(\beta_{j}^{*}\right) \quad \forall j$$
(21b)

with $W^{(*)}$ and $p^{(*)}$, let $\lambda_j^{(*)} = \lambda_j^{(**)}$, $\mu_j^{(*)} = \mu_j^{(**)} \forall j$, and go to step 2.

Lemma 4: The sequence generated by the inner iteration of Algorithm 2 is a monotonically increasing sequence and always converges. The convergence of Algorithm 2 is also guaranteed.

Proof: According to the conclusions obtained in [11], [25], [28], [29], and Lemma 3, it is easily seen that the updates of step 2 in Algorithm 2 all aim to increase the RE so that an increasing sequence is generated while the iteration is running. Since the achievable SINR region under the transmit power constraint is bounded, the RE is also bounded. Therefore, the convergence of the inner iterative of Algorithm 2 is guaranteed by the monotonic convergence theorem [35]. Combining with the conclusion obtained in [8] and [24], it is easily known that the convergence of Algorithm 2 is also guaranteed.

Note that the computational complexity of Algorithm 2 mainly lies in step 2, which needs a GP optimization operation that is used to update $\{p, \eta\}$ and an SOCP optimization operation that is used to update the beamforming vector. It is worth noting that when the number of optimization variables is relatively small, the convergence speed of the GP method is very fast, and thus, the computational complexity is very low [27], [28], [31]. As for the SOCP optimization, we know that standard convex optimization packages [33], [34] can be used to solve this problem. It is seen from (15) that the problem has $\mathcal{M} = 2 \sum_{j=1}^{K} M_j N_j$ real optimization variables, $\mathcal{N} = \sum_{j=1}^{K} N_j$ SOC constraints where each of them consists of \mathcal{M} real dimensions, and K SOC constraint where the *j*th constraint has $\mathcal{M}_i = 2M_i N_i$ real dimensions. It is known from [31] and [32] that the computational complexity of SOCP optimization in terms of the number of iterations is upper bounded by $O(\sqrt{M+K})$, and the complexity of each iteration is within the order $O(\mathcal{M}^3(\mathcal{N}+1))$. Thus, the total worstcase computational complexity of optimization problem (15) is given by $O(\sqrt{M+K}\mathcal{M}^3(N+1))$. As a result, the total computational complexity of the proposed RE optimization is given by $O(\tau_3\tau_4\sqrt{\mathcal{M}+K}\mathcal{M}^3(\mathcal{N}+1))$, where τ_3 and τ_4 , respectively, denote the operation times of step 3 and step 4.

D. Extension to Imperfect CSI

Note that the proposed RE algorithm assumes perfect CSI available at each BS; however, it is hard to realize in practical systems. It is more reasonable to use imperfect CSI in algorithm design, i.e., develop a robust RE multicell beamforming scheme. In what follows, the bounded imperfect CSI model is adopted to design the robust algorithm, which is widely adopted in the existing literature and is very useful in evaluating the developed algorithm in practical applications [36]. By decomposing the channel vector into a large-scale factor and a small-scale fading part, the actual channel can be expressed as [37], [38]

$$\boldsymbol{h}_{m,j,k} = \sqrt{\vartheta_{m,j,k}} \boldsymbol{h}_{m,j,k}^{(s)} = \sqrt{\vartheta_{m,j,k}} \left(\hat{\boldsymbol{h}}_{m,j,k}^{(s)} + \tilde{\boldsymbol{h}}_{m,j,k}^{(s)} \right) \quad (22)$$

where $\vartheta_{m,j,k}$ denotes the large-scale channel component and is assumed to be known by the BS, $\boldsymbol{h}_{m,j,k}^{(s)}$ denotes the actual small-scale fading part of the channel, $\hat{\boldsymbol{h}}_{m,j,k}^{(s)}$ denotes the nominal small-scale fading channel known by the BS, and $\tilde{\boldsymbol{h}}_{m,j,k}^{(s)}$ denotes the channel uncertainty. As commonly used in the literature, we assume that the channel uncertainty is confined within an origin-centered hyperspherical region of radius $\epsilon_{m,j,k}$, i.e., $\|\tilde{\boldsymbol{h}}_{m,j,k}^{(s)}\|^2 \leq \epsilon_{m,j,k}$, which implies that the actual channel $\boldsymbol{h}_{m,j,k}^{(s)}$ belongs to a spherical uncertainty region centered at $\hat{\boldsymbol{h}}_{m,j,k}^{(s)}$ with radius $\epsilon_{m,j,k}$.

To improve the robustness against the channel errors, the optimization problem is modified to maximize the worst-case RE over the CSI uncertainty region subject to the prescribed QoS constraints and per-BS allowed transmit power constraints, which is formulated as

$$\max_{\boldsymbol{W},\boldsymbol{p}} \sum_{j=1}^{K} \left(\frac{\kappa_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} R_{j,k}^{(\text{lb})}}{\xi \sum_{k=1}^{N_{j}} p_{j,k} + M_{j} P_{c} + P_{0}} + \frac{\beta_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} R_{j,k}^{(\text{lb})}}{\xi P_{j} + M_{j} P_{c} + P_{0}} \right)$$
(lb)

s.t.
$$\operatorname{SINR}_{j,k}^{(\operatorname{lb})} \ge \gamma_{j,k}, \sum_{k=1}^{n} p_{j,k} \le P_j, \|\boldsymbol{w}_{j,k}\| = 1 \quad \forall j,k$$
 (23)

where $R_{j,k}^{(lb)} = W \log_2(1 + \text{SINR}_{j,k}^{(lb)})$ denotes the worst-case user rate, and $\text{SINR}_{j,k}^{(lb)}$ denotes the worst-case SINR of user (j,k), which is calculated as (24), shown at the bottom of the page, [37], [38]. Following the same procedure used to solve the RE optimization problem (6), the maximization of the worst-case RE optimization problem (23) can also be

$$\operatorname{SINR}_{j,k}^{(\mathrm{lb})} = \min_{\left\{ \left\| \tilde{\boldsymbol{h}}_{m,j,k}^{(s)} \right\|^{2} \leq \epsilon_{m,j,k} \right\}} \operatorname{SINR}_{j,k}$$
$$= \frac{p_{j,k} \vartheta_{j,j,k} \left(\left| \hat{\boldsymbol{h}}_{j,j,k}^{H} \boldsymbol{w}_{j,k} \right|^{2} - \epsilon_{j,j,k} |\boldsymbol{w}_{j,k}|^{2} \right)}{\sum_{(m,n) \neq (j,k)} \left(p_{m,n} \vartheta_{m,j,k} \left(\left| \hat{\boldsymbol{h}}_{m,j,k}^{H} \boldsymbol{w}_{m,n} \right|^{2} + \epsilon_{m,j,k} |\boldsymbol{w}_{m,n}|^{2} \right) \right) + \mathcal{W}\sigma_{j,k}^{2}}$$
(24)

addressed. Note that for fixed auxiliary variables λ and μ and beamforming vector W^* , the power allocation problem can be resorted to address the following approximation problem instead of solving problem (12):

$$\max_{\boldsymbol{p},\boldsymbol{\eta}} \sum_{j=1}^{K} \left(\widetilde{f}_{j}(\boldsymbol{p},\boldsymbol{\eta}) + \frac{\beta_{j} \sum_{k=1}^{N_{j}} \alpha_{j,k} \widetilde{R}_{j,k}}{\xi P_{j} + M_{j} P_{c} + P_{0}} \right)$$

s.t. $\operatorname{SINR}_{j,k}^{(\operatorname{lb})} \geq \gamma_{j,k}, \operatorname{SINR}_{j,k}^{(\operatorname{lb})} \geq \eta_{j,k}, \sum_{k=1}^{N_{j}} p_{j,k} \leq P_{j} \quad \forall j,k$
 $(1 - \omega_{j,k}) \overline{\eta}_{j,k} \leq \eta_{j,k} \leq (1 + \omega_{j,k}) \overline{\eta}_{j,k} \quad \forall j,k.$
(25)

Similarly, let p^* and η^* be the obtained solution to problem (25) with fixed λ , μ , and W^* via the SCALE method; thus, the beamforming vector can be obtained by solving the following weighted sum power minimization problem:

$$\min_{\{\widetilde{\boldsymbol{w}}_{j,k}\}} \sum_{j=1}^{K} \sum_{k=1}^{N_j} \lambda_j \mu_j \|\widetilde{\boldsymbol{w}}_{j,k}\|^2$$

s.t. $\overline{\text{SINR}}_{j,k}^{(\text{lb})} \ge \eta_{j,k}^*, \sum_{k=1}^{N_j} \|\widetilde{\boldsymbol{w}}_{j,k}\|^2 \le P_j \quad \forall j,k$ (26)

where $\overline{\text{SINR}}_{j,k}^{(\text{lb})}$ is defined as (27), shown at the bottom of the page. Note that the optimal solutions are insensitive to any phase shift. In other words, if $\tilde{w}_{j,k}$ is an optimal solution, then $\tilde{w}_{j,k}e^{j\theta}$ is also an optimal solution as such phase shifts do not alter the objective or the constraints of problem (26). Therefore, we restrict ourselves to beamforming vectors in which $\hat{h}_{j,j,k}^H \tilde{w}_{j,k} \ge 0, \forall j, k$, i.e., each has a nonnegative real part and a zero imaginary part. Introducing auxiliary variables $t_{j,k}$, (26) is further reformulated as follows:

$$\min_{\{\widetilde{\boldsymbol{w}}_{j,k}\}} \sum_{j=1}^{K} \sum_{k=1}^{N_j} \lambda_j \mu_j \|\widetilde{\boldsymbol{w}}_{j,k}\|^2$$

s.t. $\mathfrak{h}\left(\{\widetilde{\boldsymbol{w}}_{j,k}\}\right) \leq \sqrt{\vartheta_{j,j,k}} \hat{\boldsymbol{h}}_{j,j,k}^H \widetilde{\boldsymbol{w}}_{j,k} \quad \forall j,k$
$$\sum_{k=1}^{N_j} \|\widetilde{\boldsymbol{w}}_{j,k}\|^2 \leq P_j \quad \forall j$$
(28)

where $\mathfrak{h}(\{\tilde{w}_{j,k}\})$ is defined as (29), shown at the bottom of the page. It is easily known that problem (28) is a classical SOCP problem that can be solved with some powerful optimization tools [33], [34]. As a result, the proposed Algorithm 2 can be extended to realize robust RE coordinated beamforming by calculating the value of SINR with robust SINR expression defined in (24) and solving problems (25) and (26) instead of problems (14) and (15) at step 2 in Algorithm 2, respectively.

IV. SIMULATION RESULTS

Here, we investigate the performance of the proposed multicell beamforming algorithm via numerical simulations. We consider a cooperative cluster of K = 3 hexagonal adjacent cells, where each BS j is equipped with M_j transmit antennas and serves N_j single-antenna users in cell j. The cell radius is set to be 500 m, and each user is at least 400 m from its serving BS. The channel vector $\boldsymbol{h}_{m,j,k}$ from BS mto user (j,k) is generated based on the formulation $h_{m,j,k} \triangleq$ $\sqrt{\theta_{m,j,k}} h_{m,j,k}^w$, where $h_{m,j,k}^w$ denotes the small-scale fading part and is assumed to be Gaussian distributed with zero mean and identity covariance matrix, and $\theta_{m,j,k}$ denotes the large-scale fading factor, which, in decibels, is given as $10\log_{10}(\theta_{m,j,k}) = -38\log_{10}(d_{m,j,k}) - 34.5 + \eta_{m,j,k}$, where $\eta_{m,j,k}$ represents the lognormal shadow fading with zero mean and 8-dB standard deviation [39]. The circuit power per antenna is $P_c = 30$ dBm, and the basic power consumed at the BS is $P_0 = 40$ dBm [5]. As for the power constraints, we assume that each BS has the same power constraint over the whole bandwidth. The noise figure is 9 dB. The weighted factor $\alpha_{i,k}$ is set to unit for any j and k. We assume that all elements in the weighted factor vector β have a uniform value. The noise variance $\sigma_{i,k}^2 = -174$ dBm/Hz and the occupied bandwidth W = 10 MHz. The inefficiency factor of power amplifier ξ is set to be unit. The convergence threshold $\varsigma = 10^{-4}$. For comparison, the performance of the power minimization algorithm under the same target user rate requirement and transmit power constraints is simulated. In all our simulation figures, the values of EE/SE/RE are all in units of 10M. In our simulations, the target $\gamma_{j,k}$, $\forall j, k$, is set to be the value of SINRs achieved by normalized random beamforming vectors and the random power allocation satisfied the per-BS power constraints.

$$\overline{\text{SINR}}_{j,k}^{(\text{lb})} = \frac{\vartheta_{j,j,k} \left(\left| \hat{\boldsymbol{h}}_{j,j,k}^{H} \widetilde{\boldsymbol{w}}_{j,k} \right|^{2} - \epsilon_{j,j,k} |\widetilde{\boldsymbol{w}}_{j,k}|^{2} \right)}{\sum\limits_{(m,n) \neq (j,k)} \left(\vartheta_{m,j,k} \left| \hat{\boldsymbol{h}}_{m,j,k}^{H} \widetilde{\boldsymbol{w}}_{m,n} \right|^{2} + \epsilon_{m,j,k} |\widetilde{\boldsymbol{w}}_{m,n}|^{2} \right) + \mathcal{W}\sigma_{j,k}^{2}}$$
(27)

$$\mathfrak{h}\left(\{\widetilde{\boldsymbol{w}}_{j,k}\}\right) = \sqrt{\eta_{j,k}^* \left(\sum_{(m,n)\neq(j,k)} \left(\vartheta_{m,j,k} \left| \widehat{\boldsymbol{h}}_{m,j,k}^H \widetilde{\boldsymbol{w}}_{m,n} \right|^2 + \epsilon_{m,j,k} |\widetilde{\boldsymbol{w}}_{m,n}|^2\right) + \mathcal{W}\sigma_{j,k}^2}\right) + \vartheta_{j,j,k}\epsilon_{j,j,k} |\widetilde{\boldsymbol{w}}_{j,k}|^2$$
(29)



Fig. 1. Convergence trajectory of the proposed solution, $M_j = 4$, $N_j = 2$, $P_j = 46 \text{ dBm } \forall j, \beta = 1$.



Fig. 2. Corresponding SE and EE versus β , $M_j = 4$, $N_j = 2$, $P_j = 46 \text{ dBm} \forall j$.

Fig. 1 shows the convergence trajectory of the inner loop and the outer loop of Algorithm 2 for several random channel realizations. As shown in the upper subfigure, an increasing sequence of the objective value of problem (9) is generated with the iterative running of step 2 of Algorithm 2. This is consistent with the conclusion obtained in Lemma 4. It also means that the convergence of the inner loop of Algorithm 2 can be guaranteed by the monotonic boundary sequence theorem. The results shown in the lower subfigure also demonstrate that Algorithm 2 always converges to a stable point after a limited number of iterations.

Fig. 2 plots the average EE and SE of the proposed resourceefficient algorithm over 1000 random channel realizations varying with the weighted factor β . It is easily seen that the corresponding EE decreases with increasing β while the corresponding SE increases with increasing β . This is because



Fig. 3. Corresponding SE and EE versus β , $M_j = 4$, $N_j = 2$, $P_j = 46 \text{ dBm } \forall j$.

increasing β leads to more emphasis on SE, and hence, more power is allocated for maximizing the SE. The results also show that the corresponding SE and EE remain almost unchanged when the weighted factor β tends to zero where RE optimization focuses on optimizing EE or tends to infinity where RE optimization focuses on maximizing SE. Note that the corresponding EE gradually decreases with increasing β while the corresponding SE gradually increases with increasing β , as shown in Fig. 3. These results imply that the tradeoff between SE and EE can be achieved by adjusting the value of the weighted factor β to satisfy the requirements of network operators.

Fig. 4 shows the average SE and EE versus the maximum transmit power constraint with different β over 1000 random channel realizations. It is easy to see that in the lower transmit-power region, the impact of β on the SE and EE is marginal,



Fig. 4. Corresponding SE and EE versus transmit power constraints, $M_j = 4$, $N_j = 2$, $\beta_j = 1 \forall j$.



Fig. 5. Corresponding RE versus transmit power constraints, $M_j = 16$, $N_j = 2$, $\beta_j = 1 \forall j$.

and a very close SE and EE performance is generated by Algorithm 2 for different values of β . However, in the hightransmit-power region, the corresponding EE decreases with increasing transmit power constraint, particularly for large β . This is due to the fact that the RE optimization prefers to maximize the SE and generate larger SE for large β , as verified in the average SE curve in Fig. 4. Similar results can be found in [40], which studies the weighted EE sum maximization problem subject to certain individual user rate requirements. However, we note that the change trend of the corresponding EE is distinct from that of the EE achieved by the existing algorithm, which only aims to maximize the system EE instead of taking into account the EE–SE tradeoff [5]–[8].

Fig. 5 shows the average RE performance of the proposed algorithm versus the maximum transmit power constraint over 1000 random channel realizations. One can see that the corresponding average RE increases with increasing transmit power constraints in the lower-transmit-power region, whereas it decreases with increasing transmit power constraints in the hightransmit-power constraint region. Similarly, for a fixed number



Fig. 6. Corresponding RE versus number of transmit antennas. $\beta = 1$.



Fig. 7. Corresponding RE versus transmit power constraints under imperfect channels, $M_j = 16$, $N_j = 2$, $\beta_j = 1 \forall j$.

of served users, the corresponding average RE increases with an increasing number of transmit antennas when the number of transmit antennas is small, whereas it decreases with an increasing number of transmit antennas when the number of transmit antennas is larger than a certain number of transmit antennas, as shown in Fig. 6. For example, the optimal number of transmit antennas is 16 in our simulation scenarios. However, when the ratio between the number of transmit antennas and the number of served users is fixed, the corresponding average RE increases with increasing number of transmit antennas. It is interesting to note that these observations are similar to the results obtained in [19], i.e., multiplexing to many users rather than beamforming to a single user and increasing the number of service antennas can benefit RE and obtained a tradeoff between SE and EE.

Fig. 7 shows the average worst-case RE performance of the proposed algorithm versus the maximum transmit power constraint over 1000 random imperfect channels given by (22) with different values of the hyperspherical region of radius $\epsilon_{m,j,k} = \epsilon \forall m, j, k$. In the legend, NonRobust Algorithm and Robust Algorithm denote the proposed RE optimization algorithm and its generalized algorithm to imperfect CSI, respectively. As expected, the results show that the proposed algorithm suffers an increasing performance loss with the radius of the channel uncertainty region enlarging, particularly in the high-transmit-power region. It is also observed that the proposed robust algorithm achieves an obvious performance gain over the nonrobust algorithm in all cases. This confirms the effectiveness of our robust design.

V. CONCLUSION

In this paper, we have used the RE as a new system metric for the EE-SE tradeoff in multicell multiuser multipleinput-single-output downlink systems. To address the NP-hard optimization problem under consideration, an effective algorithm has been proposed by the joint use of the fractional programming, successive convex approximation, geometry programming, and SOCP methods. The convergence of our proposed algorithm has been analyzed based on the monotonic boundary theorem and the fractional programming theory. In addition, the developed algorithm has been further extended to consider the imperfect CSI using the worst-case design. Numerical results have demonstrated the effectiveness of the proposed multicell coordinated beamforming scheme to achieve a nearoptimal RE. In the future, we shall focus on the design of distributed RE transmission schemes with low computational complexity and low exchange overhead for coordinated multicell multiuser MIMO systems.

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