2023 IEEE 24th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC) | 978-1-6654-9626-1/23/$31.00 ©2023 IEEE | DOI: 10.1109/SPAWC53906.2023.10304508

Beam Position Design for Low-latency LEO Satellite Communications with Beam Hopping

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***Abstract*—Since the latency of LEO satellite communications with beam hopping is important and mainly determined by the number of beam positions (BPs), we investigate the BP design problem, aiming at minimizing the number of BPs subject to a predefined requirement on the radius of BP. A low-complexity user density-based BP design (LCUD-BPD) scheme is proposed, where the original problem is decomposed into two subproblems, with the first one to find the sparsest user and the second one to determine the corresponding best BP. In particular, for the second subproblem, a user selection and smallest BP radius (USSBR) algorithm is proposed, where the nearby users are sequentially selected until the constraint of the given BP radius is no longer satisfied. These two subproblems are iteratively solved until all the users are selected. Simulation results have shown that the proposed scheme can substantially reduce the complexity with little performance sacrifice compared to the existing methods.**

***Index Terms*—beam position (BP) design; low earth orbit (LEO); low latency; satellite communications.**

I. Introduction

Low earth orbit (LEO) satellite constellations are capable of providing full-time communication services without blind zones, and therefore play an important role for space-terrestrial interconnection [1]. Compared with geosynchronous earth orbit satellites and medium earth orbit satellites, LEO satellites are superior in several aspects, including power loss, propagation delay and launch cost, which makes them dominant in today's commercial satellite communications [2]. Some new-generation large-scale LEO satellite constellations have recently been deployed, represented by SpaceX, OneWeb, Telesat and Kuiper. By introducing multibeam precoding to these satellites, the energy efficiency of satellite communications can be further improved [3], [4].

The latency that is an important aspect of LEO satellite communications with beam hopping (BH), is mainly measured by average packet queueing delay during the transmission from the satellites to the users. In particular, the average packet queueing delay is typically determined by the number of beam positions (BPs) [5]. To reduce the number of BPs, we need to use as less BPs as we can to cover more users, under the constraint of maximum BP radius. Aiming at minimizing the number of BPs subject to a predefined range on the radius of BP, a p-center method is presented in [5], while a heuristic method is proposed in [6] to maximize the average data rate of satellites as well as reducing the number of BPs. Since the computational complexity is an important issue in LEO satellite communications, we consider to substantially reduce the computational complexity with little performance sacrifice.

In this paper, aiming at minimizing the number of BPs subject to a predefined requirement on the radius of BP, a low-complexity user density-based BP design (LCUD-BPD) scheme is proposed. To reduce the computational complexity, the original problem is decomposed into two subproblems, where the first subproblem is to find the sparsest user and the second one is to determine the corresponding best BP. In particular, for the second subproblem, a user selection and smallest BP radius (USSBR) algorithm is proposed to determine the best BP, where the nearby users are sequentially selected until the constraint of the given BP radius is no longer satisfied. We iteratively solve these two subproblems until all the users are selected.

The rest of this paper is organized as follows. The considered system model for satellite communications is given in Section II. The BP design problem together with the proposed LCUD-BPD scheme is presented in Section III. Simulation results are provided in Section IV. Finally, Section V concludes this paper.

Notations: Symbols for matrices (upper case) and vectors (lower case) are in boldface. $(\cdot)^{\mathrm{T}}$, $|\cdot|$, $\|\cdot\|_2$, $\mathbb{C}$, $\mathbb{R}$, $\mathbb{N}$ and $\mathcal{O}(\cdot)$ denote the transpose, absolute value, ℓ_2-norm, set of complex numbers, set of real numbers, set of positive integers, order of complexity, respectively. $[\boldsymbol{a}]_n$, $[\boldsymbol{A}]_{n,:}$, $[\boldsymbol{A}]_{:,m}$ and $[\boldsymbol{A}]_{n,m}$ denote the nth entry of vector $\boldsymbol{a}$, the nth row of matrix $\boldsymbol{A}$, the mth column of matrix $\boldsymbol{A}$, and the the entry on the nth row and mth column of matrix $\boldsymbol{A}$, respectively.

II. System Model

We consider downlink satellite communications, where a LEO satellite is used to serve K randomly distributed ground users, as shown in Fig. 1. The satellite is equipped with a phased antenna array, which can form a number of spot beams and flexibly change the beam direction and beamwidth. The users are divided into M BPs based on their geographical locations, where $1 \leq M \leq K$. The number of users covered by the mth BP for $m = 1, 2, \ldots, M$ is denoted as N_m, satisfying $N_1 + N_2 + \ldots + N_M = K$. The BPs are illuminated in different BH time slots, where the BPs illuminated in the same time slot are illustrated in the same color. For the users in the same BP, the multicast transmission is adopted [7].

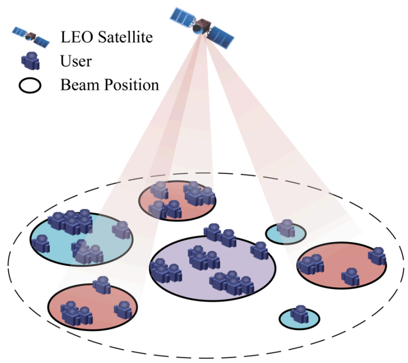


Fig. 1. Illustration of LEO satellite communications.

Since the non-line-of-sight (NLOS) channel components are usually much weaker than the line-of-sight (LOS) channel components for satellite communications, we neglect the NLOS channel components [6]. The channel vector between the satellite and the users in the mth BP, for $m = 1, 2, \ldots, M$, is denoted by $\boldsymbol{h}_m \in \mathbb{C}^{N_m}$. The kth entry of $\boldsymbol{h}_m$, for $k = 1, 2, \ldots, N_m$, can be expressed as

$$[\boldsymbol{h}_m]_k = \alpha_{m,k}\sqrt{\left(\frac{\lambda}{4\pi d_{m,k}}\right)^2 G^{\mathrm{r}}_{m,k} G^{\mathrm{t}}_{m,k}}, \tag{1}$$

where $\alpha_{m,k}$ and λ denote the power attenuation of the kth user in the mth BP and the wavelength, respectively. The distance between the satellite and this user is denoted by $d_{m,k}$. The receive antenna gain of this user is denoted as $G^{\mathrm{r}}_{m,k}$. Assuming that the hopping beams are circular, we denote the transmit antenna gain as $G^{\mathrm{t}}_{m,k}$, which can be expressed as

$$G^{\mathrm{t}}_{m,k} = g_m\left(\frac{J_1(\mu_{m,k})}{2\mu_{m,k}} + 36\frac{J_3(\mu_{m,k})}{\mu^3_{m,k}}\right), \tag{2}$$

where $J_1(\cdot)$ and $J_3(\cdot)$ denote the Bessel function of the first kind and the third kind, respectively [8]. According to the convention, we define $\mu_{m,k} \triangleq 2.07123\sin(\theta_k)/\sin(\vartheta_m)$, where θ_k and ϑ_m denote the off-axis angle of the kth user and the 3dB-gain beamwidth of the mth BP in unit of angle, respectively. When $\theta_k = 0$, the achieved maximum transmit antenna gain is $g_m = \eta\beta^2\pi^2/\vartheta^2_m$, where η denotes the antenna efficiency. β is a constant, which equals 65 for phased antenna array.

To ensure the quality of service (QoS) for the users, we assume that each beam points at the center of each BP and the users in the same BP are all located in the mainlobe of the beam [9]. Let $\boldsymbol{r} \in \mathbb{R}^M$ denote a vector including the radius of the M BPs. The radius of the mth BP, for $m = 1, 2, ..., M$, can be expressed as

$$[\boldsymbol{r}]_m = S\tan\left(\frac{\vartheta_m}{2}\right), \tag{3}$$

where S denotes the height of satellite. For each BP, its channel coefficient is defined to be that between the satellite and the worst user as [6]

$$\zeta_m \triangleq \min_k \left[|\boldsymbol{h}_m|\right]_k. \tag{4}$$

Then the capacity of the LEO satellite communications for the mth BP, $m = 1, 2, ..., M$, can be written as

$$C_m = B_{\mathrm{tot}}\log_2\left(1 + \frac{P_{\mathrm{tot}}|\zeta_m|^2}{\sigma b B_{\mathrm{tot}}}\right), \tag{5}$$

where B_{tot}, P_{tot}, σ and b denote the total bandwidth, the total transmit power, the noise power spectral density and the maximum number of the hopping beams in each time slot, respectively. We set $b \leq M$. To maximize the spectral efficiency, each BP uses the whole bandwidth, while the total transmit power is uniformly allocated to the b hopping beams [5].

III. Beam Position Design

In this section, we will investigate the BP design for low-latency LEO satellite communications.

A. Problem Formulation

The latency measured by average packet queueing delay, can be expressed as a function of M as

$$f(M) = \frac{\Delta t \sum_{m=1}^{M} C_m \sum_{i=1}^{M}\sum_{j=1}^{N_m} A_j/C_i}{2b\sum_{m=1}^{M}\sum_{j=1}^{N_m} A_j}, \tag{6}$$

where Δt and A_j denote the duration of one time slot and the average packet arrival rate for the jth user, respectively [5]. Note that Δt and b are limited by the hardware conditions and therefore are predefined. The parameter A_j is randomly generated from real set [20,200].

To reduce the latency, we aim to minimize the number of BPs, resulting in large radius of BPs. On one hand, larger radius of BPs is capable of covering more users. On the other hand, it will lead to the transmit antenna gain of users becoming smaller, which may cause the QoS of the users to be unsatisfied in the worst case. Therefore, we aim at minimizing the number of BPs subject to a predefined requirement on the radius of BP, where the predefined requirement is essentially determined by the QoS of the users. We define $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$ and $\mathcal{M} \triangleq \{1, 2, \ldots, M\}$. Then the BP design problem can be formulated as

$$\min_{\boldsymbol{X},\boldsymbol{W},\boldsymbol{r}} \quad M, \tag{7a}$$

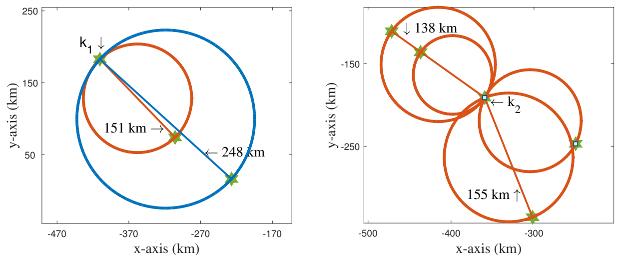
$$\text{s.t.} \quad \sum_{m=1}^{M}[\boldsymbol{X}]_{m,k} \geq 1,\ \forall k \in \mathcal{K}, \tag{7b}$$

$$[\boldsymbol{r}]_m \geq r_{\min},\ \forall m \in \mathcal{M}, \tag{7c}$$

$$[\boldsymbol{r}]_m \leq r_{\max},\ \forall m \in \mathcal{M}, \tag{7d}$$

$$\left\|[\boldsymbol{W}]_{m,:} - [\boldsymbol{U}]_{k,:}\right\|_2 \leq [\boldsymbol{r}]_m, \quad \forall m, k \in \{m, k \mid [\boldsymbol{X}]_{m,k} = 1\}, \tag{7e}$$

where $\boldsymbol{W} \in \mathbb{R}^{M\times 2}$ and $\boldsymbol{U} \in \mathbb{R}^{K\times 2}$ denote the two-dimensional coordinates at the centers of the M BPs and the two-dimensional coordinates of the K users, respectively. We assume that the locations of the users are known to the satellite [6]. For a binary matrix $\boldsymbol{X} \in \mathbb{N}^{M\times K}$ indicating the relationship between a user and its covering BP, if the



(a) Sparsely populated area (b) Densely populated area

Fig. 2. Sparsely populated area and densely populated area with different numbers of candidate BPs.

kth user is covered by the mth BP, $[\boldsymbol{X}]_{m,k} = 1$; otherwise $[\boldsymbol{X}]_{m,k} = 0$. Constraint (7b) states that to achieve the full coverage of users by the LEO satellite, each user must be covered by at least one BP. Constraint (7c) states that the minimum radius of BPs, which is denoted as $r_{\min} \triangleq 0.443\lambda S/D$, is limited by hardware conditions, including wavelength λ, the height of satellite S and the diameter of phased antenna array D [10]. Constraint (7d) states that the maximum radius of BPs, denoted as $r_{\max}$, is a predefined requirement essentially determined by the QoS of the users. Constraint (7e) states that for each BP, its radius should be sufficient to cover all users belonging to this BP.

B. LCUD-BPD Scheme

Once an optimal solution, i.e., an optimal M is obtain in (7), there may be multiple solutions for $\boldsymbol{r}$, $\boldsymbol{X}$ and $\boldsymbol{W}$. In fact, since constraints (7d) and (7e) only give the upper and lower bounds of the BP radius, multiple solutions of $\boldsymbol{r}$ could be figured out. But intuitively, we prefer smaller $\boldsymbol{r}$, as smaller $\boldsymbol{r}$ leads to larger transmit antenna gain and larger SNR of the users.

In the following, the LCUD-BPD scheme will be proposed to solve (7), where the original BP design problem is decomposed into two subproblems. The first subproblem is to find the sparsest user, and the second one is to determine the corresponding best BP. For the second subproblem, the USSBR algorithm will be proposed. These two subproblems will be iteratively solved until all the K users are selected.

1) First Subproblem to Find the Sparsest User: The users are randomly distributed in the coverage area of the LEO satellite. Therefore, we define a distance matrix $\boldsymbol{D} \in \mathbb{R}^{K\times K}$, where the distance between the ith user and the jth user can be expressed as

$$[\boldsymbol{D}]_{i,j} = \left\|[\boldsymbol{U}]_{i,:} - [\boldsymbol{U}]_{j,:}\right\|_2, \ \forall i, j \in \mathcal{K}. \tag{8}$$

To better describe the distribution of the K users, we define the user density vector as $\boldsymbol{\rho} \in \mathbb{R}^K$, which can be expressed as

$$[\boldsymbol{\rho}]_k \triangleq \sum_{i=1}^{K} [\boldsymbol{D}]_{k,i}, \ \forall k \in \mathcal{K}. \tag{9}$$

Algorithm 1 USSBR algorithm.

Require: $\mathcal{L}$, $\boldsymbol{U}$, $r_{\max}$, $r_{\min}$, $k_{\rm s}$.
Ensure: $\boldsymbol{z}_{\rm s}$, $\omega_{\rm s}$, $\mathcal{J}_m$, $\mathcal{I}$.

```
Initialize z_s, ω_s, J_m and I via (12).
while I ≠ ∅ do
    Obtain [D]_{m_2,:} via (8).
    Find a user k̃ via (13) and (14).
    Update J_m via (15).
    Obtain z, ω using randomized algorithm.
    if ω ≤ r_max then
        Update z_s and ω_s via (16).
    else
        Update J_m and I via (17).
        Break.
    end if
end while
```

If $[\boldsymbol{\rho}]_k$ is small, the kth user is located in a densely populated area; otherwise, the kth user is located in a sparsely populated area.

For different users, the number of candidate BPs is different. Fig. 2 shows the number of candidate BPs for different users, with the maximum radius $r_{\max} = 100$ km. For user k_1 in a sparsely populated area, there is only one candidate BP in red solid line that covers user k_1 and his nearby users. The BP in blue dashed line does not satisfy the $r_{\max}$ constraint. For user k_2 in a densely populated area, there are four candidate BPs. For the users in sparsely populated areas, the number of candidate BPs is small, which means that they are less likely to be covered by the same BP as other users. Therefore, the BP design scheme starts from the sparsest user, which is denoted as $k_{\rm s}$ and can be determined by

$$k_{\rm s} = \arg\max_{k\in\mathcal{L}} [\boldsymbol{\rho}]_k, \tag{10}$$

where $\mathcal{L}$ denotes the set of remaining users during the total iterative BP design and is initialized to be $\mathcal{K}$ as

$$\mathcal{L} \leftarrow \mathcal{K}. \tag{11}$$

2) Second Subproblem to Determine the Best BP: Once the sparsest user $k_{\rm s}$ is determined, we need to determine the corresponding best BP.

When designing the mth BP, the center coordinates of the mth BP, the radius of the mth BP, the set including the user $k_{\rm s}$ and his nearby users covered by the mth BP, and the set of remaining users during the iterative BP design, denoted as $\boldsymbol{z}_{\rm s} \in \mathbb{R}^2$, $\omega_{\rm s}$, $\mathcal{J}_m$ and $\mathcal{I}$, respectively, are initialized by

$$\boldsymbol{z}_{\rm s} \leftarrow [\boldsymbol{U}]_{k_{\rm s},:}, \ \omega_{\rm s} \leftarrow r_{\min}, \ \mathcal{J}_m \leftarrow \{k_{\rm s}\}, \ \mathcal{I} \leftarrow \mathcal{L}. \tag{12}$$

We compute the distance between the center of the mth BP and the users in $\mathcal{I}$ via (8). Then we find a user $\tilde{k}$, which is most likely to be covered together with the users in $\mathcal{J}_m$ by the mth BP. Therefore, $\tilde{k}$ should satisfy the following constraints

$$[\boldsymbol{D}]_{m,\tilde{k}} > [\boldsymbol{r}]_m, \tag{13}$$

$$[\boldsymbol{D}]_{m,\tilde{k}} \le [\boldsymbol{D}]_{m,i}, \ \forall i \in \mathcal{I}\backslash\mathcal{J}_m. \tag{14}$$

Algorithm 2 LCUD-BPD scheme

Require: $\mathcal{K}$, $\boldsymbol{U}$, $r_{\max}$, $r_{\min}$.
Ensure: $\boldsymbol{W}$, $\boldsymbol{r}$, $\boldsymbol{X}$, M.
Initialize m, $\boldsymbol{W}$, $\boldsymbol{r}$, $\boldsymbol{X}$ via (18) and $\mathcal{L}$ via (11).
Obtain user density $\hat{\boldsymbol{\rho}}$ via (9).
while $\mathcal{L} \neq \emptyset$ **do**
  $m \leftarrow m + 1$.
  Obtain the user $k_{\rm s}$ via (10).
  Obtain $\boldsymbol{z}_{\rm s}$, $\omega_{\rm s}$, $\mathcal{J}_m$, $\mathcal{I}$ via **Algorithm** 1.
  Update $\boldsymbol{W}$, $\boldsymbol{r}$, $\mathcal{L}$ via (19) and $\boldsymbol{X}$ via (20).
end while
$M \leftarrow m$.

In fact, constraint (13) indicates that the user $\tilde{k}$ has not yet been covered by the mth BP. Constraint (14) states that except for the users that have already been covered by the mth BP, the user $\tilde{k}$ is closest to the center of the mth BP. Then we update $\mathcal{J}_m$ by

$$\mathcal{J}_m \leftarrow \mathcal{J}_m \cup \{\tilde{k}\}. \tag{15}$$

To cover all the users in $\mathcal{J}_m$, the BP with the smallest radius is determined by the randomized algorithm [11], where the obtained center coordinates and the radius of the BP are denoted as $\boldsymbol{z} \in \mathbb{R}^2$ and ω, respectively. If constraint (7d) is satisfied, i.e., $\omega \leq r_{\max}$, we update the center coordinates and the radius of the mth BP to be $\boldsymbol{z}$ and ω as

$$\boldsymbol{z}_{\rm s} \leftarrow \boldsymbol{z},\ \omega_{\rm s} \leftarrow \omega, \tag{16}$$

respectively. In this way, the nearby users are sequentially selected until the constraint of the given BP radius is not satisfied or there are no users to be selected. If constraint (7d) is not satisfied, we remove the user $\tilde{k}$ from $\mathcal{J}_m$ and update $\mathcal{I}$ by

$$\mathcal{J}_m \leftarrow \mathcal{J}_m \backslash \tilde{k},\ \mathcal{I} \leftarrow \mathcal{I} \backslash \mathcal{J}_m, \tag{17}$$

respectively.

These steps of solving the second subproblem to determine the best BP, named the USSBR algorithm, are summarized in **Algorithm** 1. The number of iterations in **Algorithm** 1 is relevant to user distribution, the number of users in $\mathcal{I}$ and the maximum radius.

Finally, by integrating the steps to solve the first and the second subproblems, we propose the LCUD-BPD scheme, which is summarized in **Algorithm** 2.

We first initialize the iteration counter, the center coordinates matrix of the M BPs, the radius vector of the M BPs, and the binary matrix indicating the relationship between a user and its covered BP as

$$m \leftarrow 0,\ \boldsymbol{W} \leftarrow \emptyset,\ \boldsymbol{r} \leftarrow \emptyset,\ \boldsymbol{X} \leftarrow \emptyset, \tag{18}$$

respectively. By obtaining the user $k_{\rm s}$ via (10), we solve the first subproblem to find the sparsest user. By obtaining $\boldsymbol{z}_{\rm s}$, $\omega_{\rm s}$, $\mathcal{J}_m$, $\mathcal{I}$ via **Algorithm** 1, we solve the second subproblem to determine the best BP. Then we update $\boldsymbol{W}$, $\boldsymbol{r}$ and $\mathcal{L}$ as

$$\boldsymbol{W} \leftarrow [\boldsymbol{W}^{\rm T}, \boldsymbol{z}_{\rm s}]^{\rm T},\ \boldsymbol{r} \leftarrow [\boldsymbol{r}^{\rm T}, \omega_{\rm s}]^{\rm T},\ \mathcal{L} \leftarrow \mathcal{I}, \tag{19}$$

respectively. To update $\boldsymbol{X}$, we first introduce a temporary zero vector $\boldsymbol{x} \leftarrow \boldsymbol{0}^K$, then set $[\boldsymbol{x}]_k \leftarrow 1$, $\forall k \in \mathcal{J}_m$ and finally update $\boldsymbol{X}$ by

$$\boldsymbol{X} \leftarrow [\boldsymbol{X}^{\rm T}, \boldsymbol{x}]^{\rm T}. \tag{20}$$

These two subproblems will be iteratively solved until all the K users are selected, i.e. $\mathcal{L} = \emptyset$. In fact, the number of iterations for **Algorithm** 2 is M.

C. Computational Complexity Analysis

It can be observed that **Algorithm** 2 needs M outer iterations. During each outer iteration, **Algorithm** 1 needs at most T_2 inner iterations, where T_2 is relevant to user distribution, the number of users in $\mathcal{I}$ and the maximum radius. During each inner iteration, the computational complexity is no higher than $\mathcal{O}(K)$ on computing the distance between the center of a BP and the K users. Therefore, the total computational complexity of **Algorithm** 2 is $\mathcal{O}\big(MT_2K\big)$.

As a comparison, the computational complexity of the existing p-center method and user grouping method is $\mathcal{O}\left(K^2(K-1)/2\right)$ [12] and $\mathcal{O}(T_{\rm u}KM)$ [6], respectively, where $T_{\rm u}$ is the number of iterations. The numerical comparisons for our proposed scheme and the existing methods will be included in Section IV.

TABLE I
SIMULATION PARAMETERS.

Symbols	Definition	Value
S	The height of satellite	1000 km
$P_{\rm tot}$	Total transmit power	100 W
$B_{\rm tot}$	Total bandwidth	240 MHz
b	Maximum number of hopping beams	4
Δt	Duration of one time slot	2 ms
$r_{\max}$	Maximum radius of BP	100 km
$r_{\min}$	Minimum radius of BP	24 km
$G^{\rm r}_{m,k}$	Receive antenna gain	40 dBi

IV. SIMULATION RESULTS

In this section, we evaluate both the performance and the computational complexity for the LCUD-BPD scheme. A LEO satellite working at 20GHz Ka band covers an area with the radius of $500{\rm km}$ on the ground [5]. For simplicity, the receive antenna gain is assumed to be the same for all users. The detailed parameters for the simulation are provided in Table I.

Fig. 3 compares the latency, which is denoted as $f(M)$ in (6), obtained by the LCUD-BPD scheme, p-center method [5] and user grouping method [6], when the number of users increases from $K = 25$ to $K = 200$. It is seen that both the LCUD-BPD scheme and the p-center method perform much better than the user grouping method. As K increases, three curves all climb. When we increase from $K = 50$ to $K = 200$, the performance reduction of the LCUD-BPD scheme over the p-center method grows from

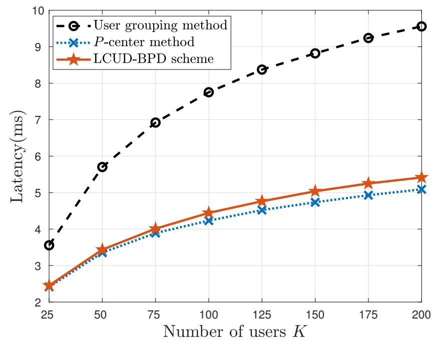


Fig. 3. Comparison of the latency for different numbers of users.

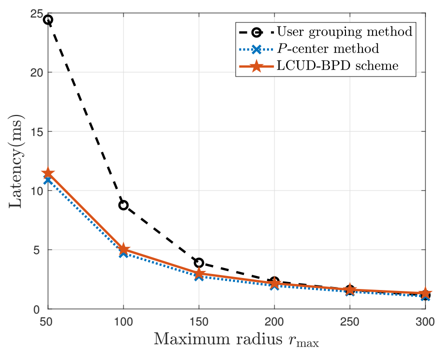


Fig. 4. Comparison of the latency for different maximum radius.

1.49% to 6.38%; however, the sacrifice of the computational complexity is much larger, i.e., the reduction in computational complexity of the LCUD-BPD scheme over the p-center method grows from 89.13% to 97.8% according to Section III-C. Moreover, we also compare the running time for the LCUD-BPD scheme and the p-center method using the same computer hardware and software. When we set $K = 200$, the running time for the LCUD-BPD scheme and the p-center method is 0.0208s and 10.8105s, respectively, which results in 99.8% reduction in computational complexity of the LCUD-BPD scheme over the p-center method.

Fig. 4 compares the latency obtained by the LCUD-BPD scheme, p-center method and user grouping method when the number of users is fixed to be $K = 150$ and the maximum radius of BPs increases from $r_{\max} = 50$ to $r_{\max} = 300$. It is seen that both the LCUD-BPD scheme and the p-center method perform better than the user grouping method. As $r_{\max}$ increases, three curves all decline and the performance gap in terms of latency between any two curves gets small. Note that if $r_{\max}$ grows to be 500, which equals the radius of the coverage area of the LEO satellite, the number of BPs obtained by three method would all be $M = 1$, resulting in the same latency for three curves.

Fig. 5 illustrates the BP design results if we set $K = 50$ and $r_{\max} = 100$. It is seen that we can get $M = 14$, where the largest and smallest radius of the designed BPs are 90.74 km and 24km, respectively. The blue and red points correspond

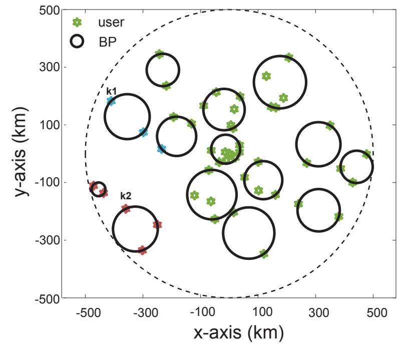


Fig. 5. Illustration of the BP design results using LCUD-BPD scheme.

to the points in Fig. 2(a) and Fig. 2(b), respectively.

V. Conclusion

In this paper, aiming at minimizing the number of BPs subject to a predefined requirement on the radius of BP, we have proposed the LCUD-BPD scheme. Simulation results have shown that the proposed scheme can substantially reduce the complexity with little performance sacrifice compared to the existing methods.

Acknowledgment

This work is supported in part by National Key Research and Development Program of China under Grant 2021YFB2900404.

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